HW#3 Due 2022/11/23

1. Derive

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \vec{u} \cdot \nabla \rho$$

and

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \vec{u} \cdot \nabla f + \vec{a} \cdot \nabla_u f$$

where ρ and f are the mass density and distribution function, respectively.

2. Show that

$$\rho \frac{d\vec{u}}{dt} = -\nabla P + \rho_c \vec{E} + \vec{J} \times \vec{B}$$

can be rewritten as

$$\frac{\partial}{\partial t}\left(\rho\vec{u} + \frac{1}{C^2}\frac{\vec{E}\times\vec{B}}{\mu_0}\right) + \nabla\cdot(\rho\vec{u}\vec{u} + \left(P + \frac{1}{2}\epsilon_0 E^2 + \frac{B^2}{2\mu_0}\right)\vec{1} - (\epsilon_0\vec{E}\vec{E} + \frac{\vec{B}\vec{B}}{\mu_0})) = 0$$

First, you need to show that $\frac{\partial}{\partial t}(\rho\vec{u}) + \nabla\cdot(\rho\vec{u}\vec{u}) = \rho\frac{d\vec{u}}{dt}$

by using $\nabla\cdot(\vec{A}\vec{B}) = (\nabla\cdot\vec{A})\vec{B} + (\vec{A}\cdot\nabla)\vec{B}$

3. Show that

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho u^2 + U_{int}\right) + \nabla \cdot \left(\left(\frac{1}{2}\rho u^2 + U_{int}\right)\vec{u} + P\vec{u} + Q\right) = \vec{J} \cdot \vec{E}$$

can be rewritten as

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho u^2 + U_{int} + \frac{1}{2}\epsilon_0 E^2 + \frac{B^2}{2\mu_0}\right) + \nabla \cdot \left(\left(\frac{1}{2}\rho u^2 + U_{int}\right)\vec{u} + P\vec{u} + Q + \frac{\vec{E} \times \vec{B}}{\mu_0}\right) = 0$$