② $$-\int_{-\epsilon}^{\epsilon} u \frac{\partial f}{\partial x} dx$$

$$= -\left(\int_{-\epsilon}^{0} u \frac{\partial f}{\partial x} dx + \int_{0}^{\epsilon} u \frac{\partial f}{\partial x} dx\right)$$

$$= -\left(u\int_{-\epsilon}^{0} \frac{\partial f}{\partial x} dx + u\int_{0}^{\epsilon} \frac{\partial f}{\partial x} dx\right)$$

$$= -\left(u_1 f\Big|_{-\epsilon}^{0} + u_2 f\Big|_{0}^{\epsilon}\right)$$

$$= -\left(u_1\left(Ae^{-\frac{u_1}{k_1}(0)} - Ae^{-\frac{u_1}{k_1}\epsilon}\right) + u_2(A-A)\right)$$

$$= -u_1\left(A - Ae^{-\frac{u_1}{k_1}\epsilon}\right) \simeq 0$$

③ $\frac{1}{3}\int_{-\epsilon}^{\epsilon} \frac{\partial u}{\partial x}\frac{\partial f}{\partial \ln p}\,dx$

$= \frac{1}{3}\left(up\frac{\partial f}{\partial p}\right)\Big|_{-\epsilon}^{\epsilon}$

$$\frac{\partial f}{\partial p} = \begin{cases} \frac{\partial A}{\partial p} e^{\frac{u_1}{\kappa_1}x}, & x<0 \\ \frac{\partial A}{\partial p}, & x \geq 0 \end{cases}$$

$= \frac{1}{3}\left(u_2 p\frac{\partial A}{\partial p} - u_1 p e^{\frac{u_1}{\kappa_1}\epsilon}\frac{\partial A}{\partial p}\right)$

$= \frac{1}{3}(u_2 - u_1)\, p\, \frac{\partial A}{\partial p}$

∵ ① + ② + ③ = 0

$$\Rightarrow -u_1 A + 0 + \frac{1}{3}(u_2 - u_1)\, p \frac{\partial A}{\partial p} = 0$$

$$\Rightarrow \frac{1}{3}(u_2 - u_1)\, p \frac{\partial A}{\partial p} = u_1 A$$

$$\Rightarrow \int_{A_0}^{A} \frac{dA}{A} = \int_{p_0}^{p} \frac{3u_1}{u_2 - u_1} \frac{dp}{p}$$

$$\frac{3u_1}{u_2 - u_1} = \frac{3\,\frac{u_1}{u_2}}{1 - \frac{u_1}{u_2}} = -\frac{3\alpha}{\alpha - 1}, \qquad \frac{u_1}{u_2} \equiv \alpha, \qquad \frac{3\alpha}{\alpha - 1} \equiv \Gamma$$

$$\Rightarrow A = A_0 \left(\frac{p}{p_0}\right)^{-\Gamma}$$

$$\therefore f(x,p) = \begin{cases} A_0 \left(\frac{p}{p_0}\right)^{-\Gamma} e^{\frac{u_1}{\kappa_1} x}, & x < 0 \\ A_0 \left(\frac{p}{p_0}\right)^{-\Gamma}, & x \geq 0 \end{cases}$$

where $\Gamma \equiv \frac{3\alpha}{\alpha - 1} = \frac{3u_1}{u_1 - u_2}$

* the solution is a power law.

* the power exponent only dep on the compression ratio.

* the stronger shocks produce the flatter (i.e. smaller Γ value) energy spectra than the weaker shocks.

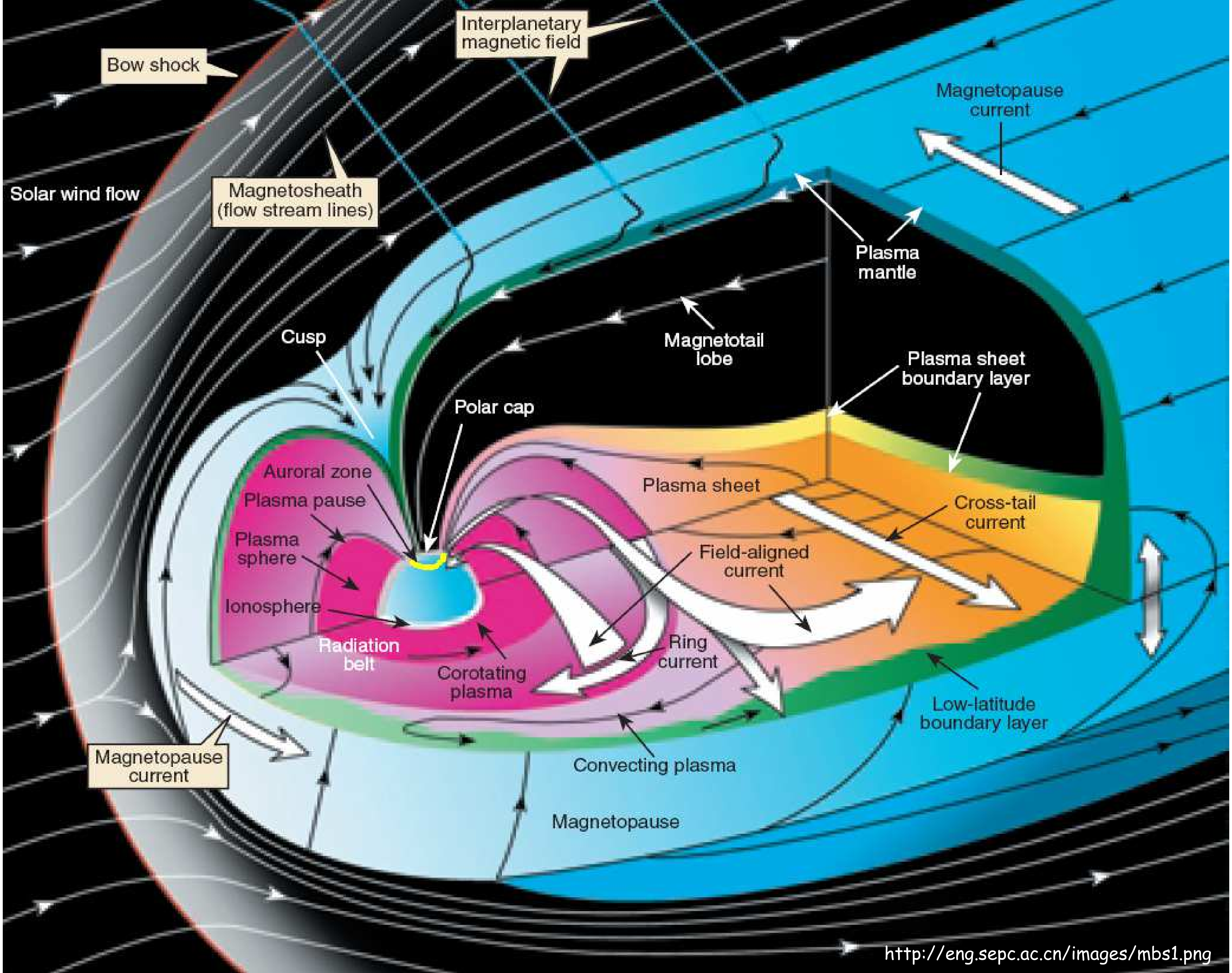
The acc. rate in diffusive shock acc. is given by

$$\frac{1}{\tau} \propto \frac{V_s^2}{K}$$

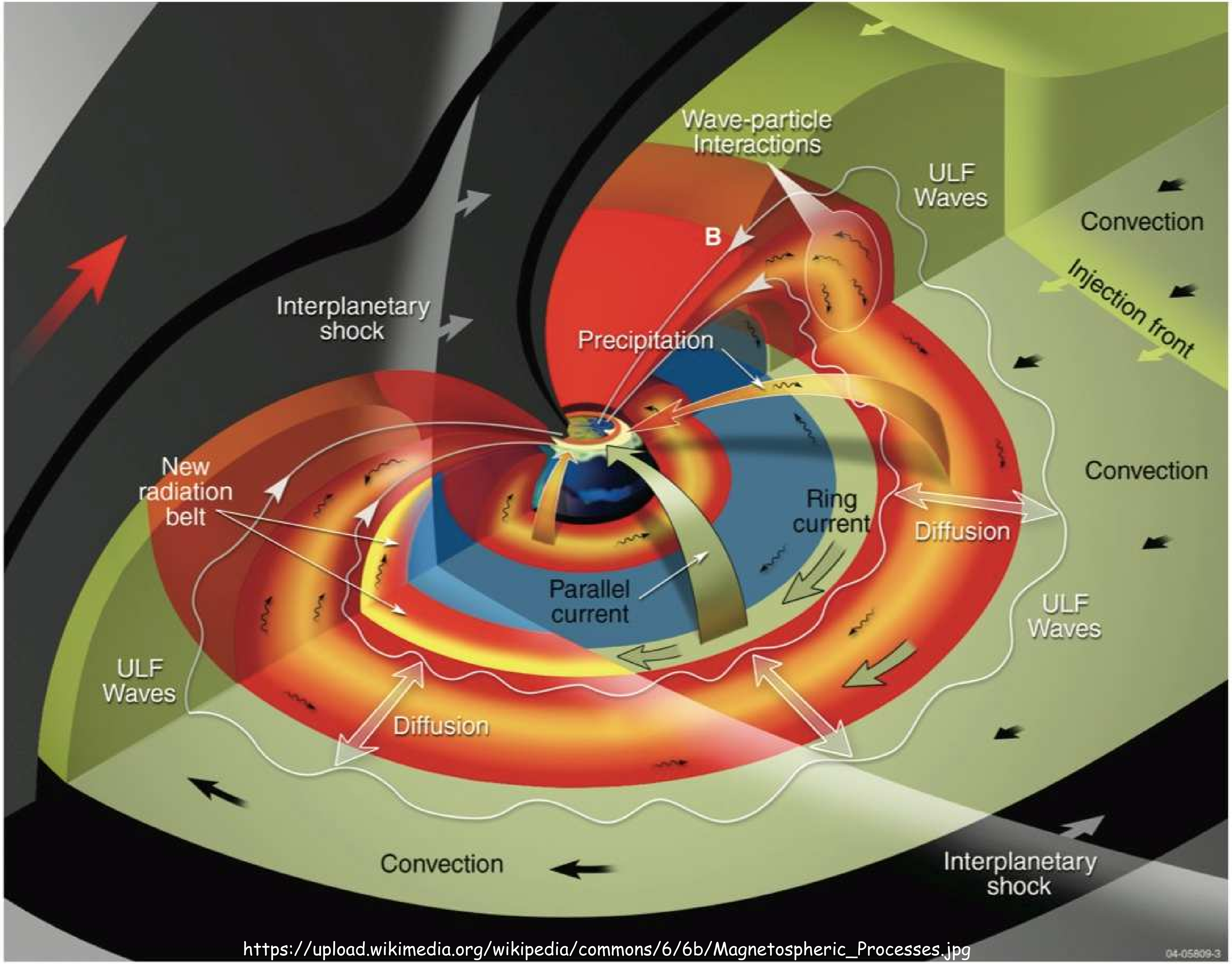
$\because K_\perp \ll K_\parallel$

$\Rightarrow$ the acc. rate is higher for a perpendicular shock.

For a given time interval, a perpendicular shock will yield a larger max. energy than a parallel shock.



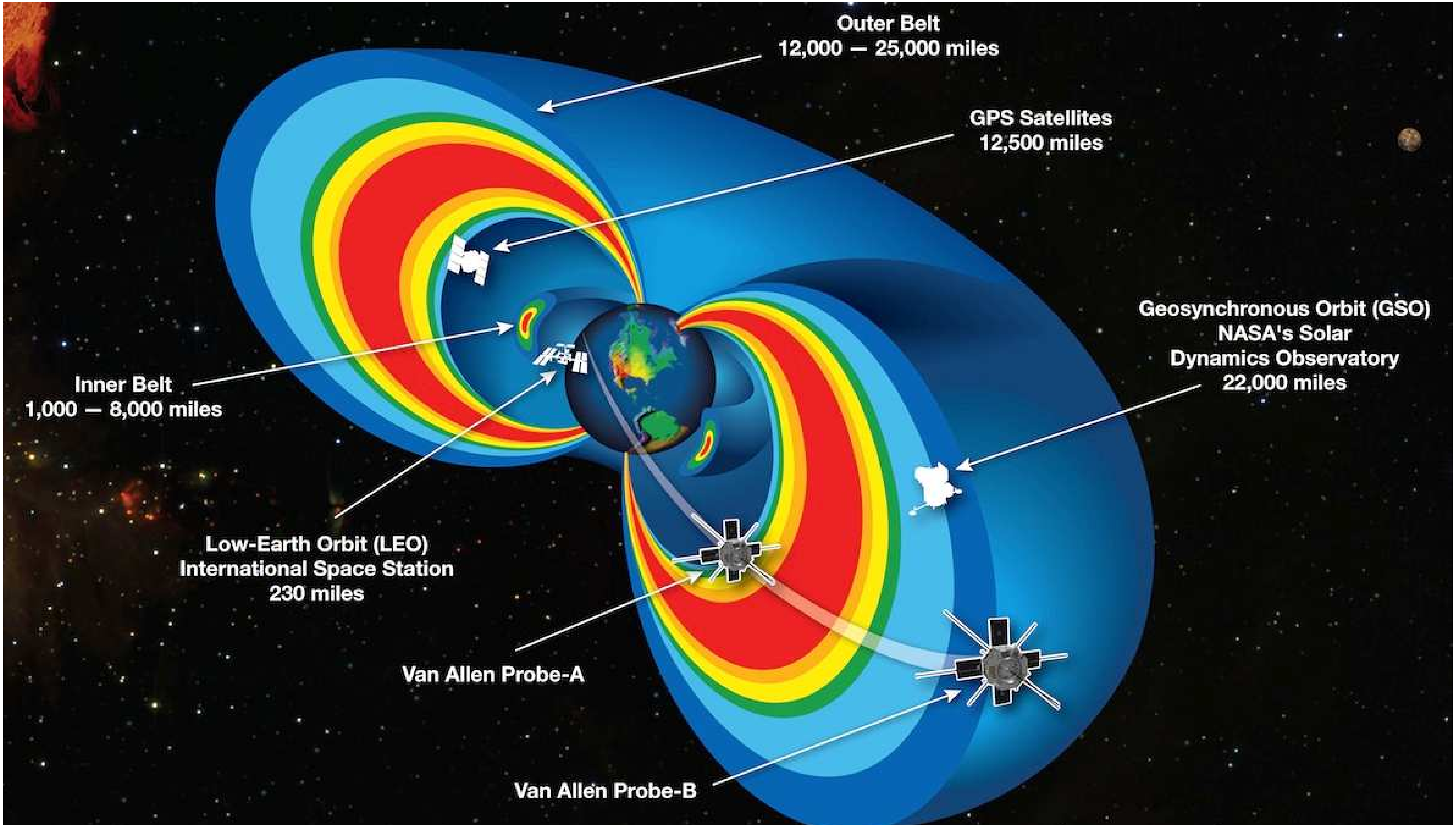
http://eng.sepc.ac.cn/images/mbs1.png



https://upload.wikimedia.org/wikipedia/commons/6/6b/Magnetospheric_Processes.jpg

- Typical temperature of ionosphere e-: ~ 1 eV
- Typical temperature of the solar wind e-: 10 eV
- Radiation Belt e-: 400 keV – 15 MeV

→How do they get accelerated to such high energies?



http://www.slashgear.com/wp-content/uploads/2013/12/radiation_belts1.jpg

Yihua Zheng (2013)

Inner Magnetosphere Challenges

Generation of waves
- →interactions between plasmas and fields

Wave properties
- →chorus, hiss, EMIC wave amplitudes, growth rates, location

Net balance between sources and losses
- →identification of all processes

Wave-particle interactions
- →energy, pitch-angle diffusion

External driving
- →solar wind, magnetosphere, and ionosphere

External driving
- →plasma sheet sources, E & B fields, diffusion rates, ionospheric outflow
- →solar wind coupling

Tuija I. Pulkkinen (2007)

Cross-Energy Coupling

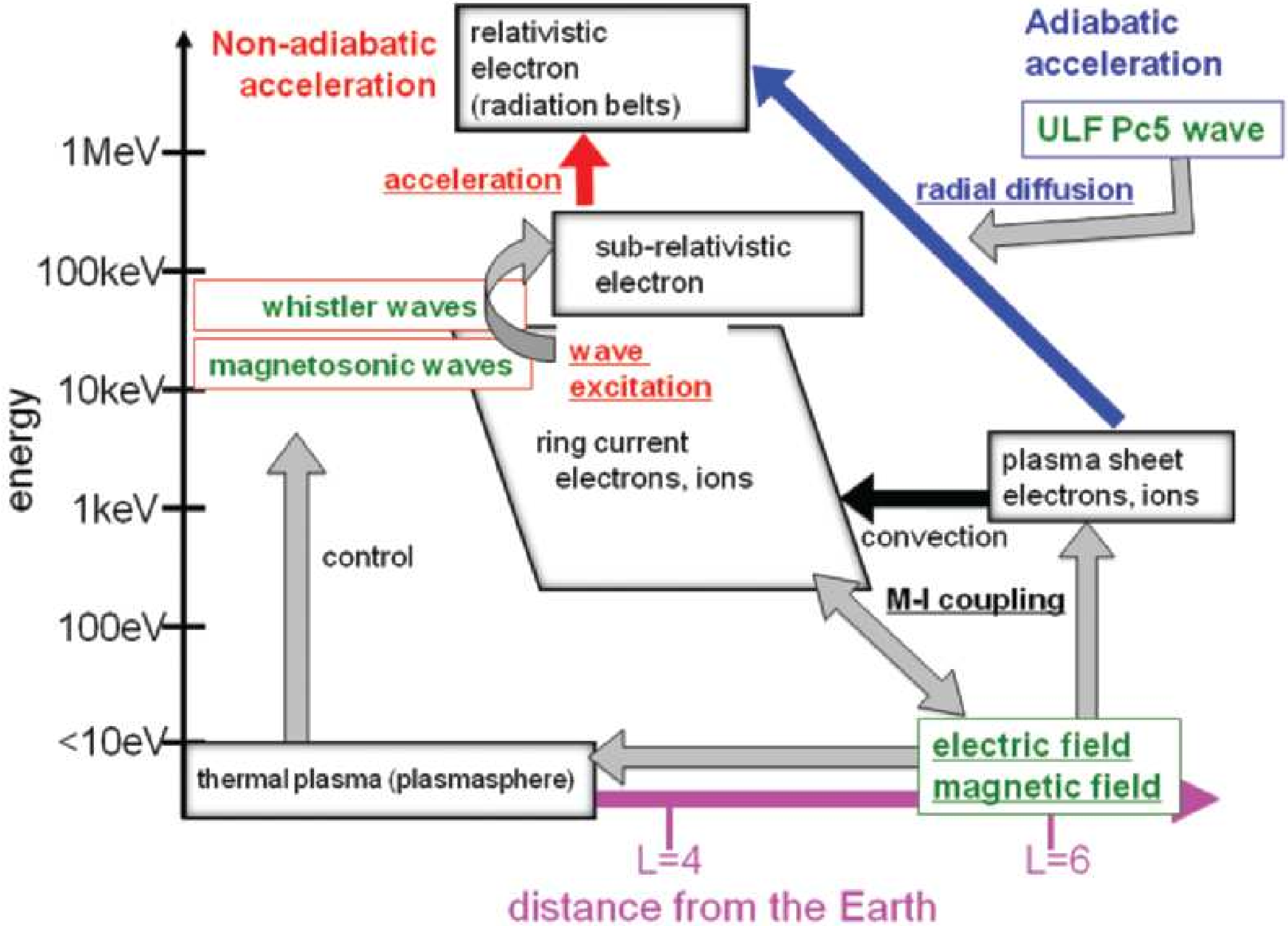


Fig. 7-2 in D. Summers, I. R. Mann, D. N. Baker, & M. Schulz (2012)

Inner Magnetosphere

Table 1. Characteristics of Inner Magnetospheric Plasma Populations

Population	Density	Temperature	Source	Composition
Plasmasphere	100s cm^{-3} to 1000s	<1 eV, maybe up to 10s of eV	Subauroral ionosphere	H^+, some He^+ and O^+
Ring current	~few cm^{-3}, up to 10s	1–400 keV	Plasma sheet (SW and iono)	H^+, O^+ in storms
Radiation belts	$\ll$1 cm^{-3}	100s of keV to MeV	Plasma sheet, SEPs, local acc.	Mostly e-, some H^+

Plasmasphere

→1-10 eV ions

→ionospheric origin

Ring current

→1-400 keV ions

→both ionospheric and solar wind origin

Outer radiation belt

→0.4-10 MeV electrons

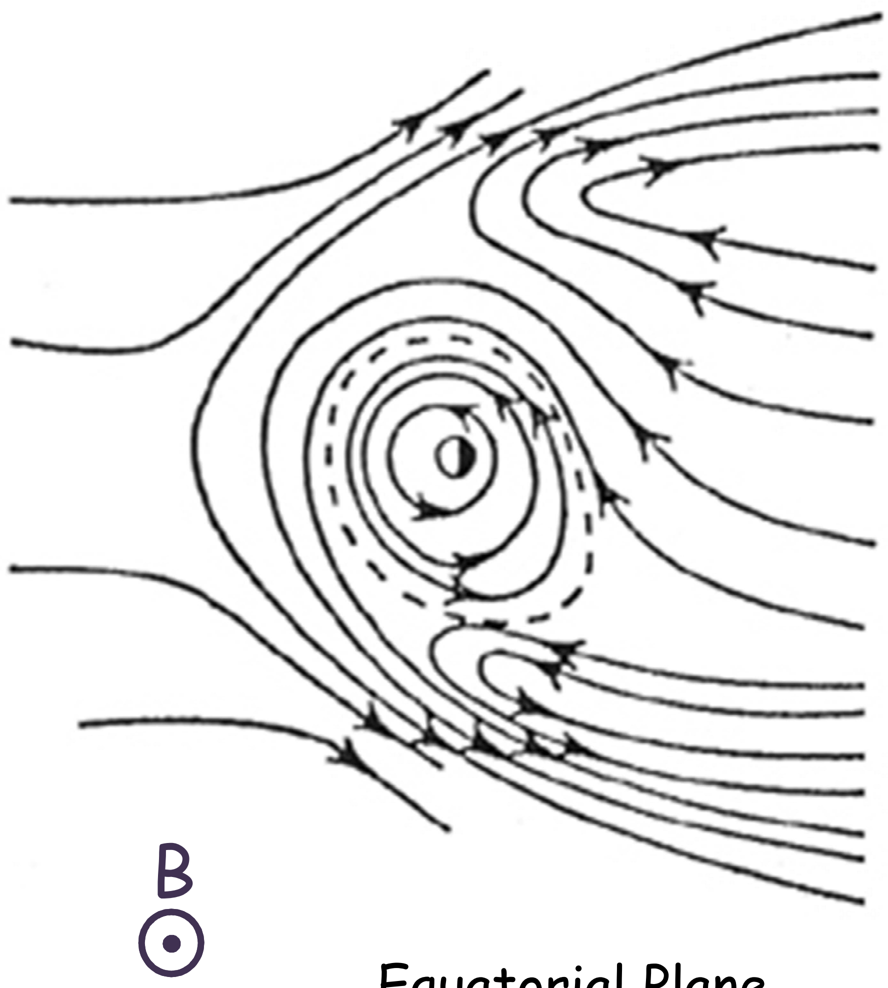
→magnetospheric origin

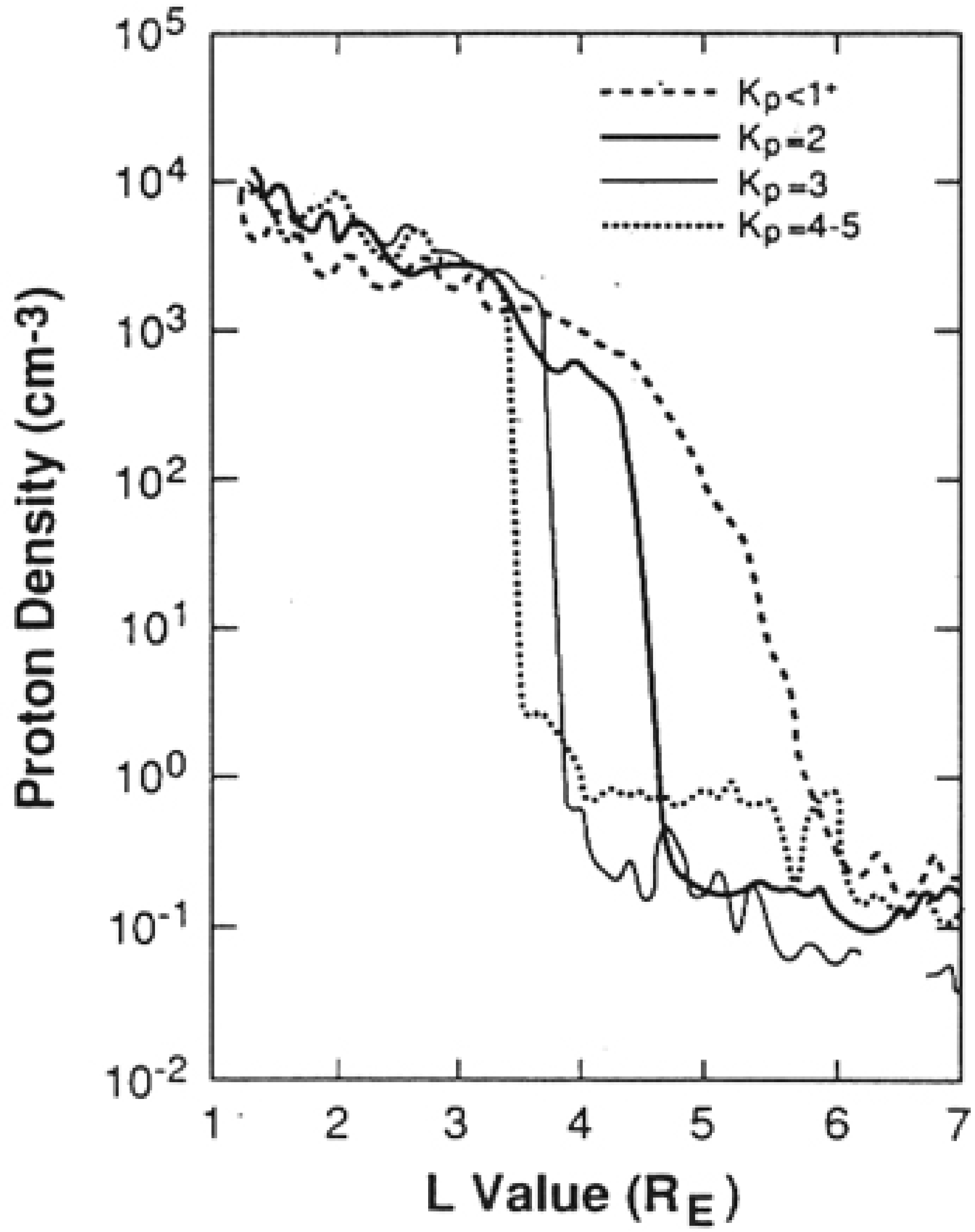
"The inner magnetosphere is a major player of space weather."

Liemohn (2006), Yihua Zheng (2013)

Plasmasphere

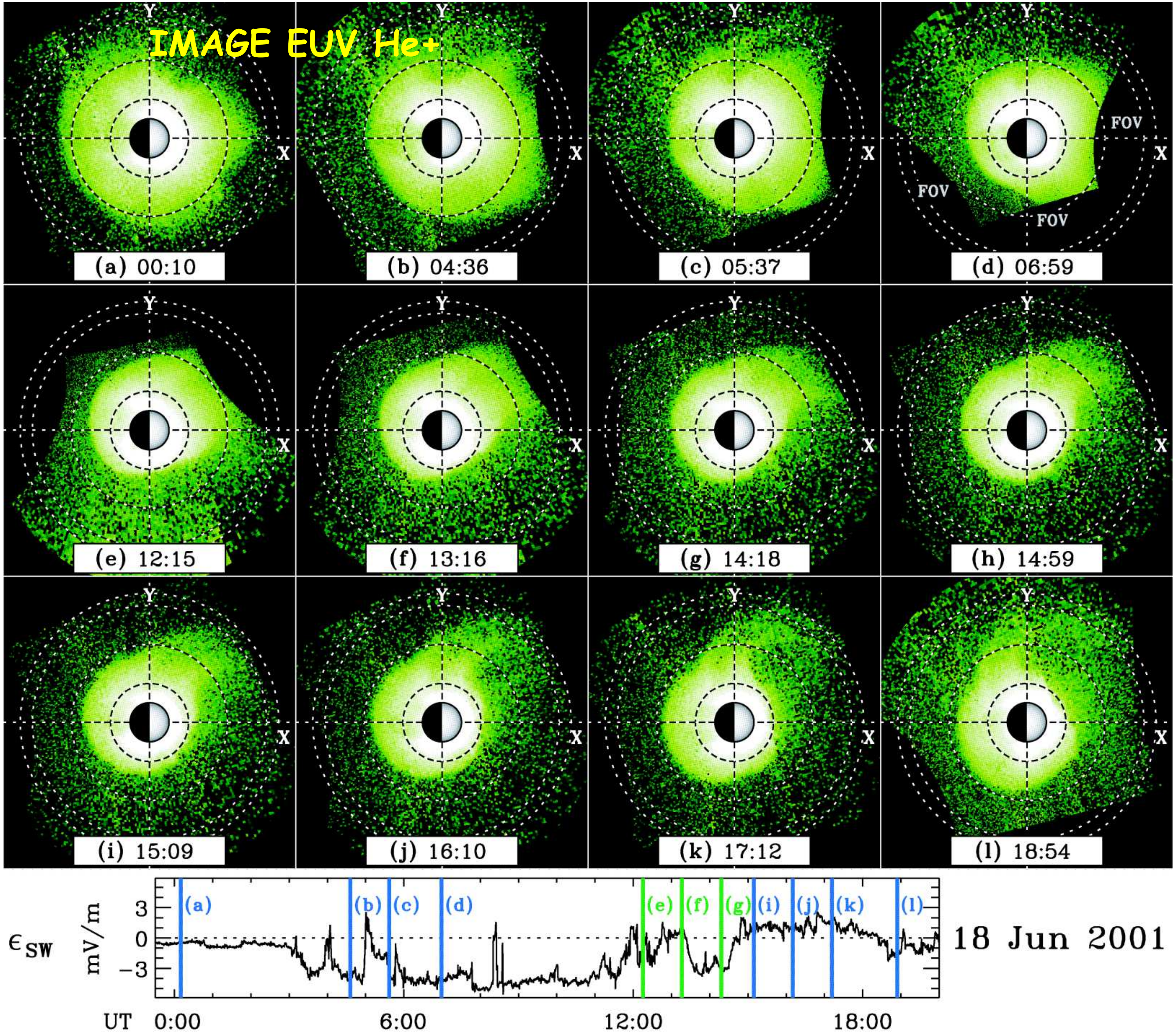
- **Cold**: Less than 1 eV, maybe up to 10 eV
- **Dense**: 100s-1000s cm^{-3}, lower out near geos.
- **Ionospheric source**
- **Mostly Protons**: often-quoted composition, 77% H+, 20% He+, and 3% O+
- **E-field dominated**: spatial extent governed by magnetospheric electric field time history, B is also important
- Dominates the mass density of the inner magnetosphere





http://www.ss.ncu.edu.tw/~lyu/lecture_files/IntroSpace.html

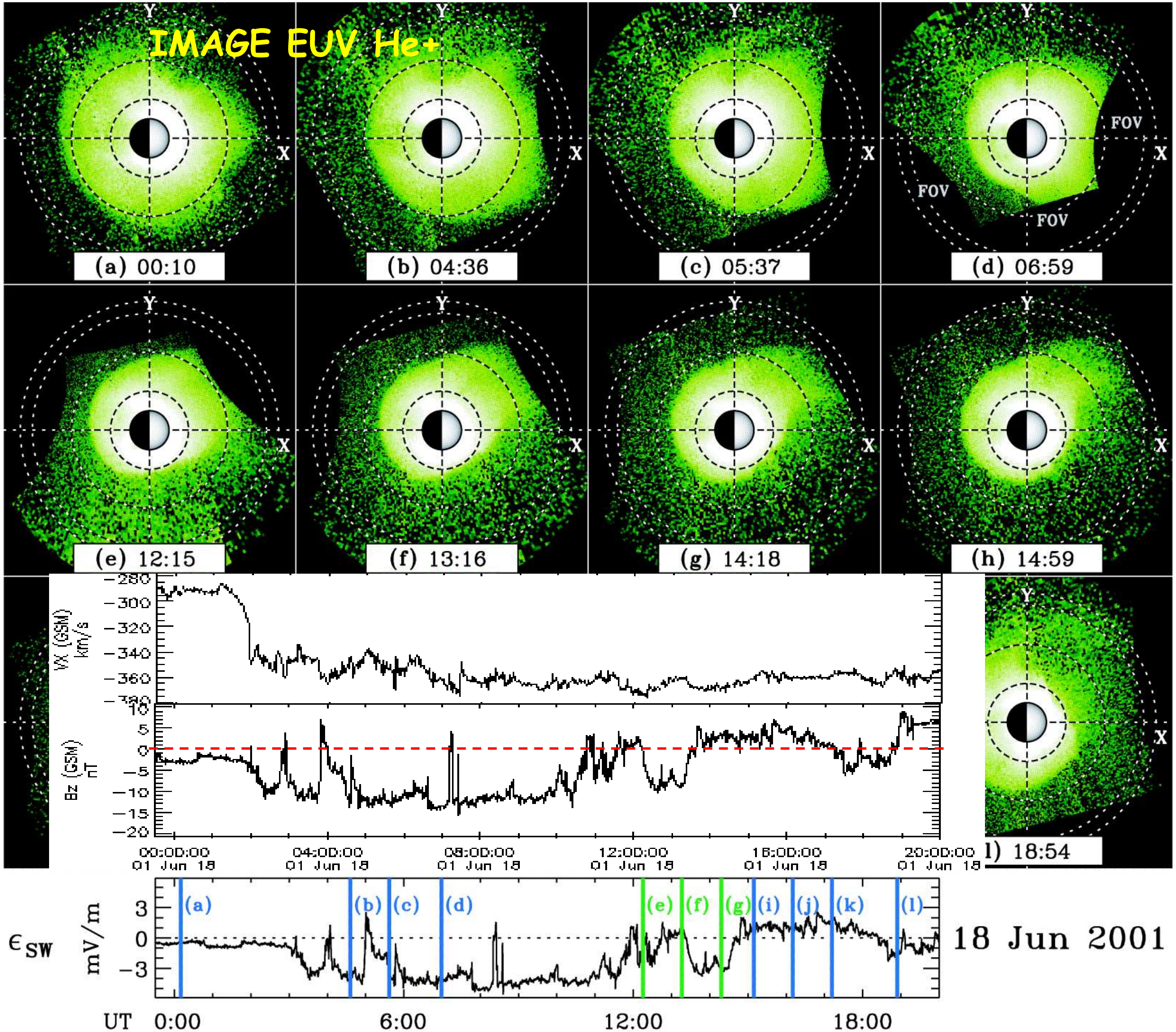
Fig 10.15 in M. G. Kivlson & C. T. Russell (1995)



Responds to solar wind driving:

• Erosion of the plasmasphere • Formation/evolution of a drainage plume

Yihua Zheng (2013)



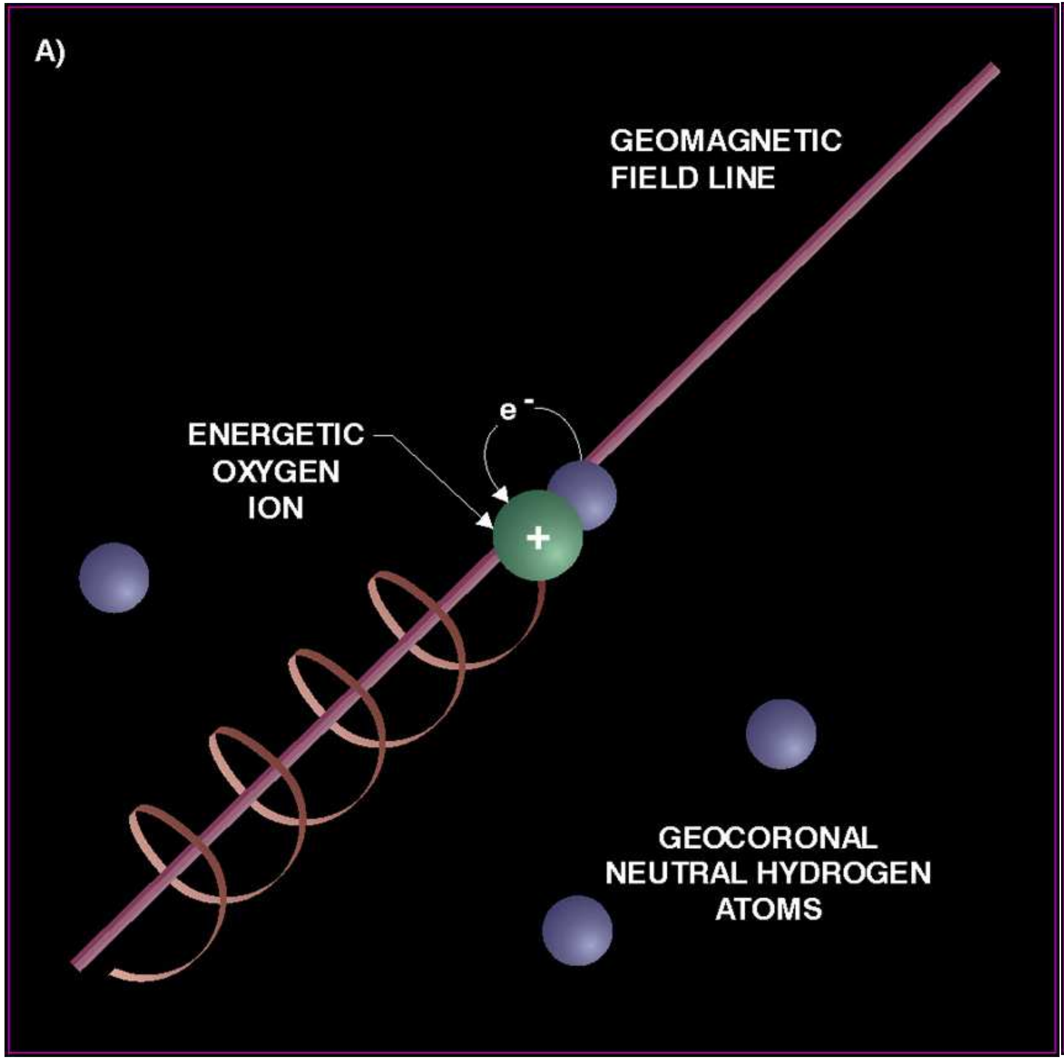
Responds to solar wind driving:

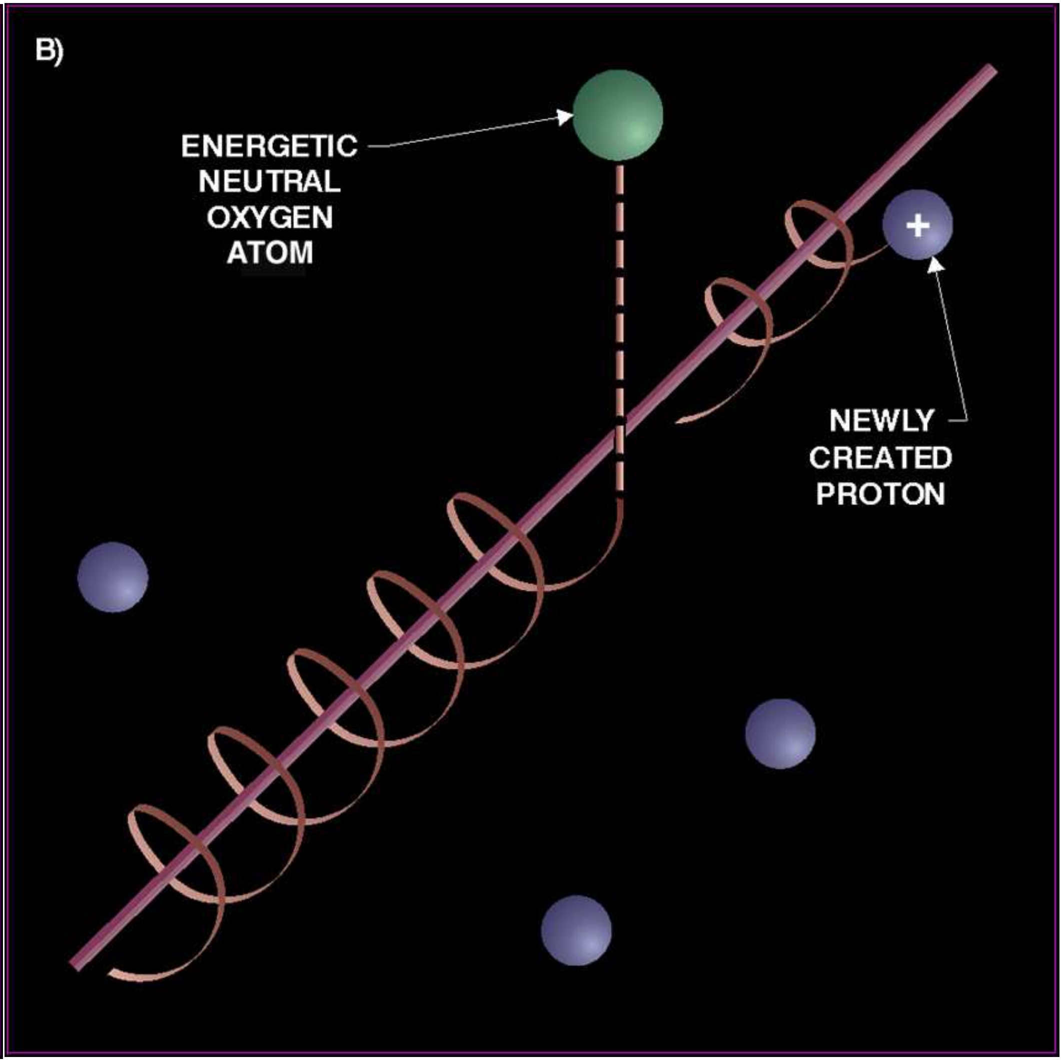
- Erosion of the plasmasphere • Formation/evolution of a drainage plume

Yihua Zheng (2013)

Ring Current

- **Hot**: 1-400 keV
- **Tenuous**: quiet, 1 cm^{-3}; active, 10s cm^{-3}
- **Plasma sheet**: source is near-Earth magnetotail (from solar wind or ionosphere)
- **Mostly Protons**: During big storms, O+ can dominate
- **Complicated Drift**: E-field, B-field, gradient-curvature terms
- Dominates the energy density of inner magnetosphere

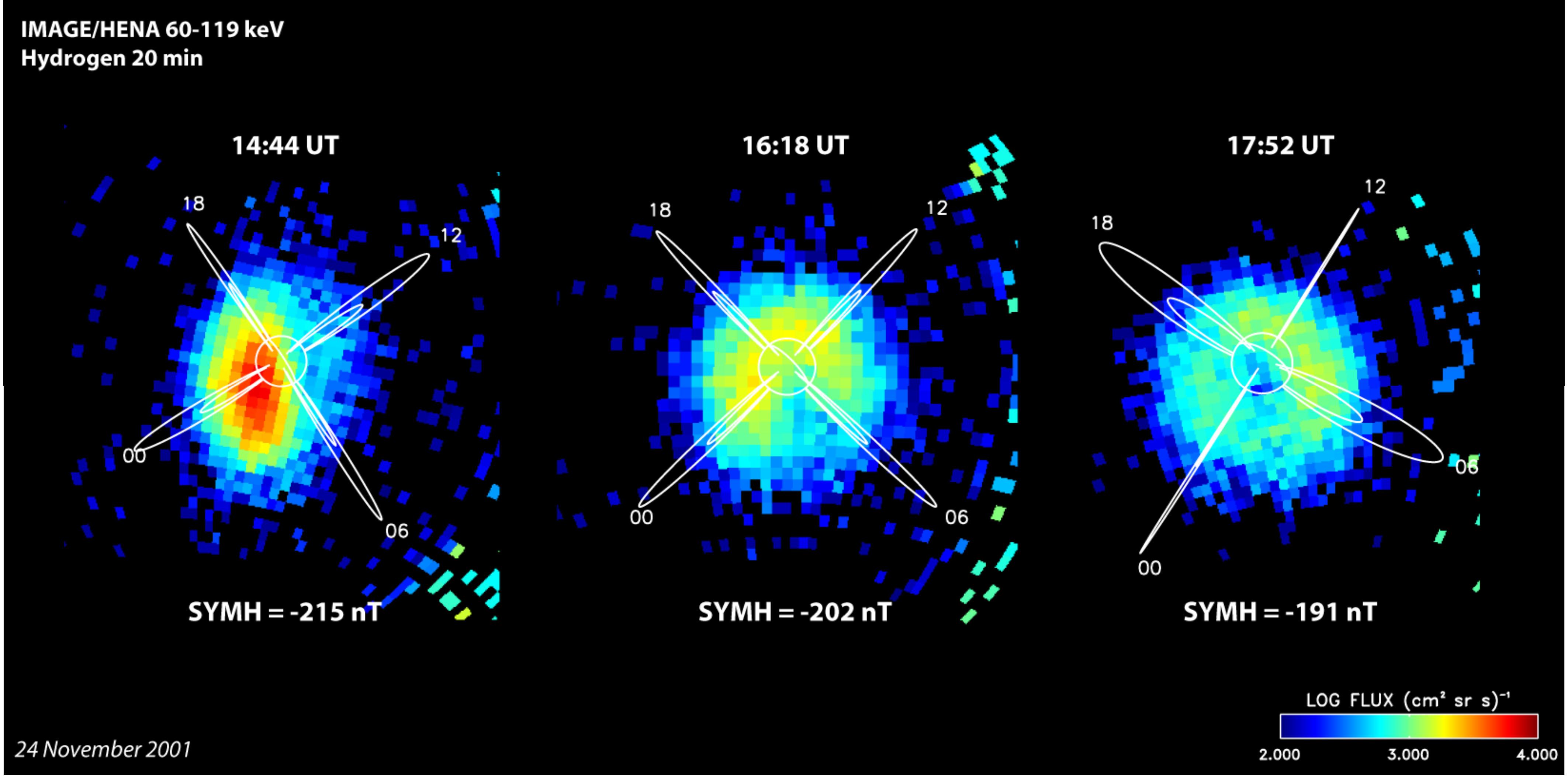




a magnetically trapped ion captures an electron from a neutral hydrogen atom

creating an energetic neutral atom (ENA) that is no longer trapped

ENA is an important tool of imaging the ring current.



- Highly responsive to changes in solar-wind conditions
- Change morphology in a couple of hours

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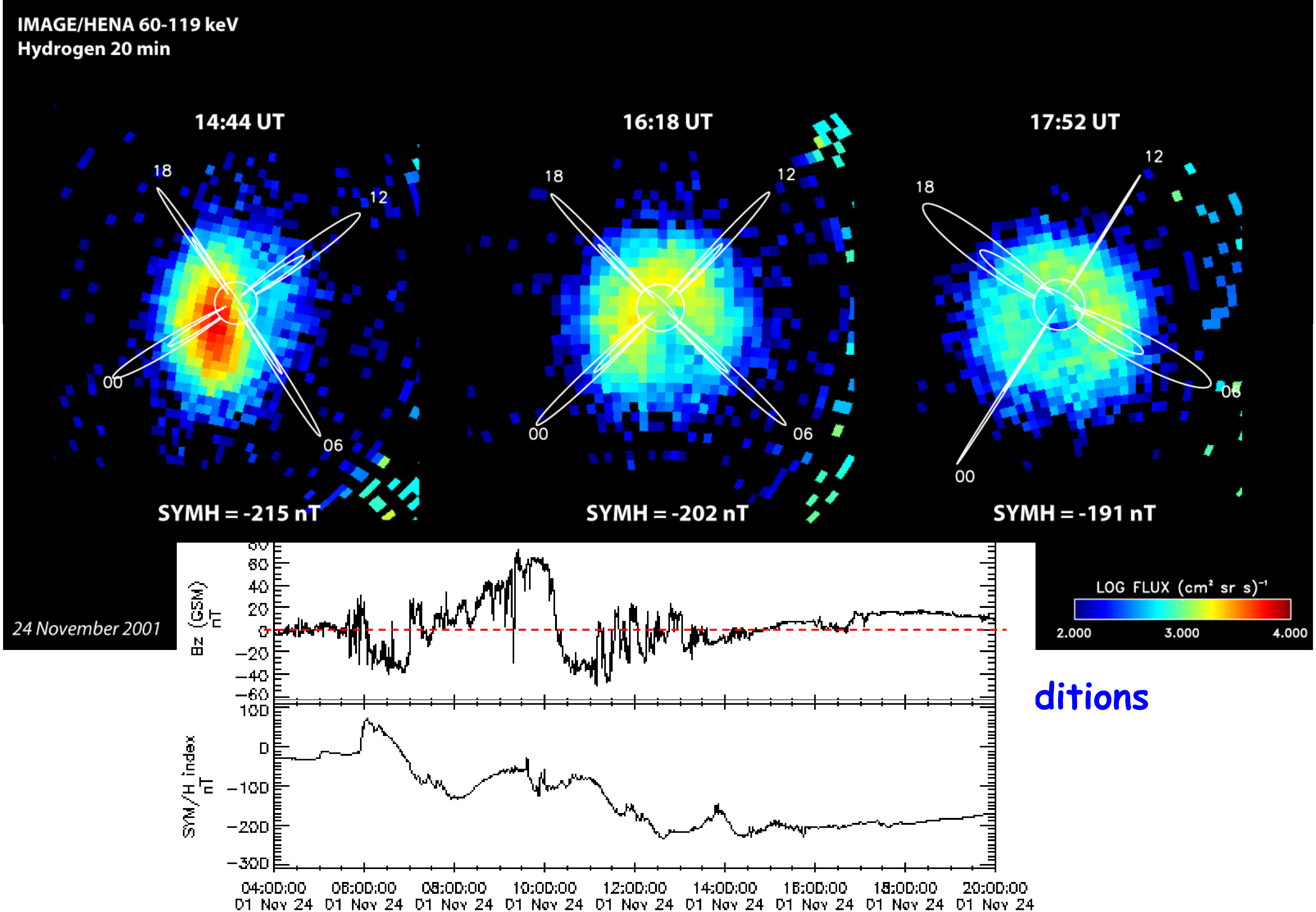

ditions

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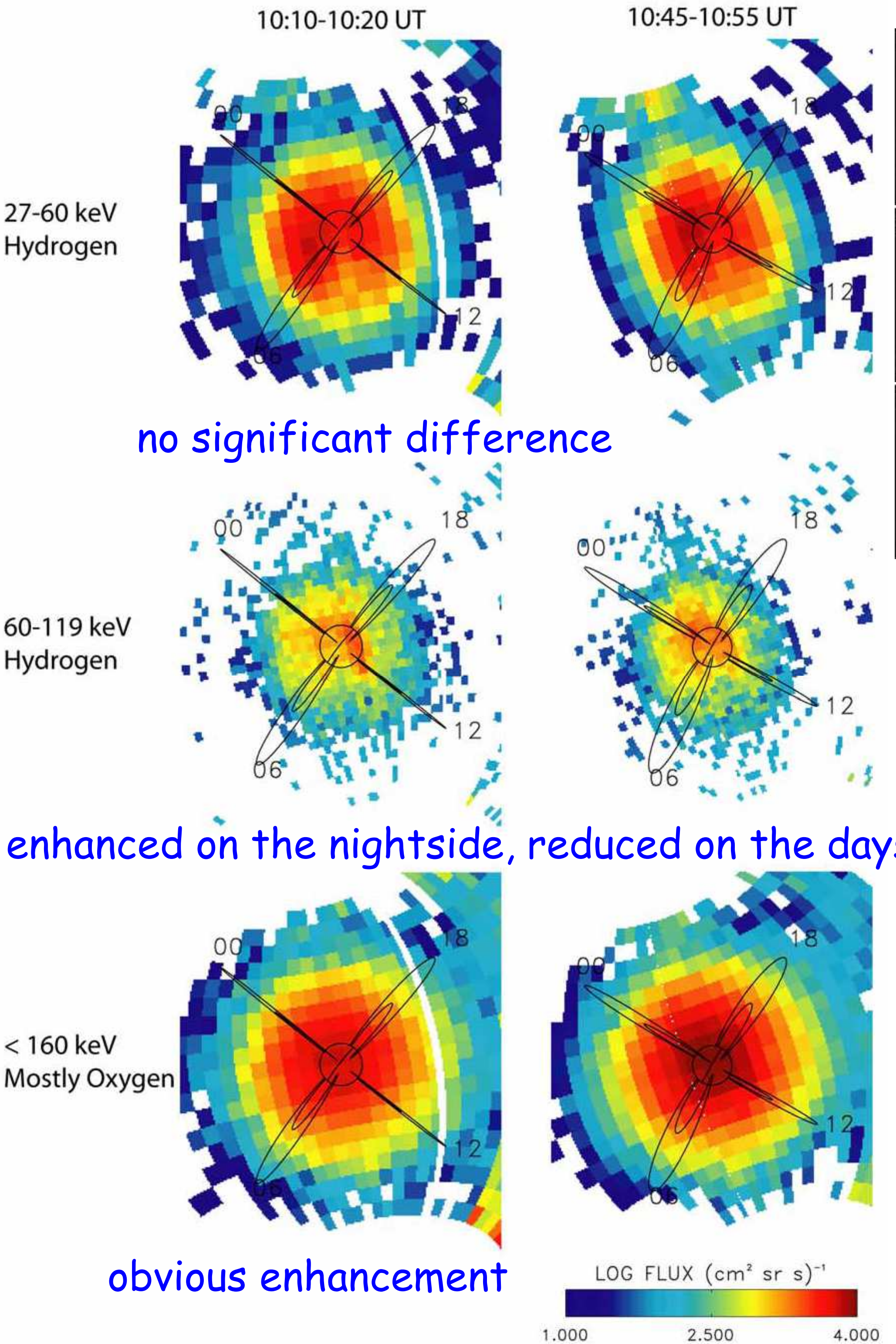
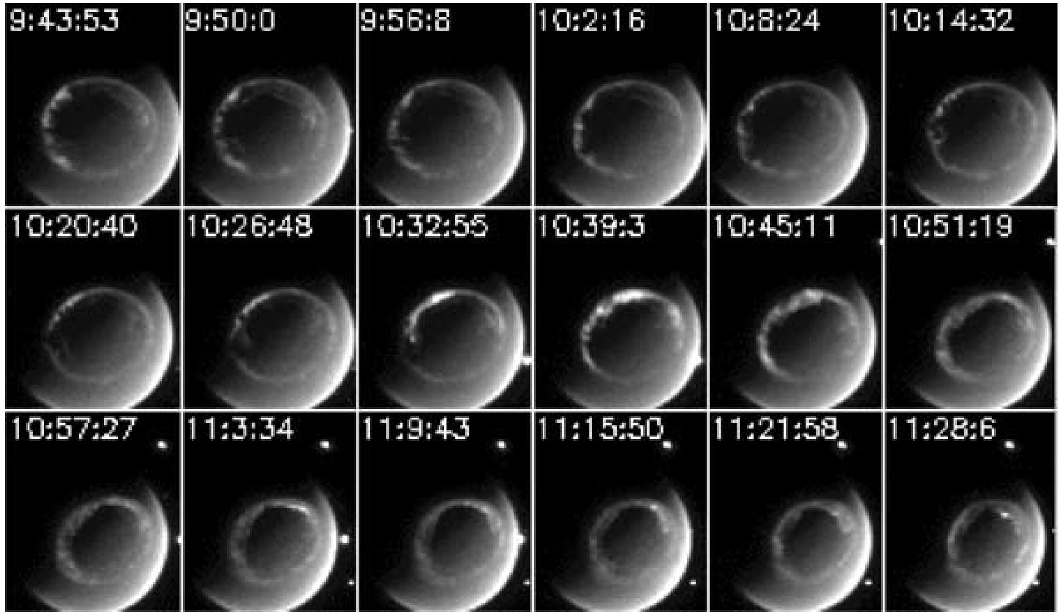
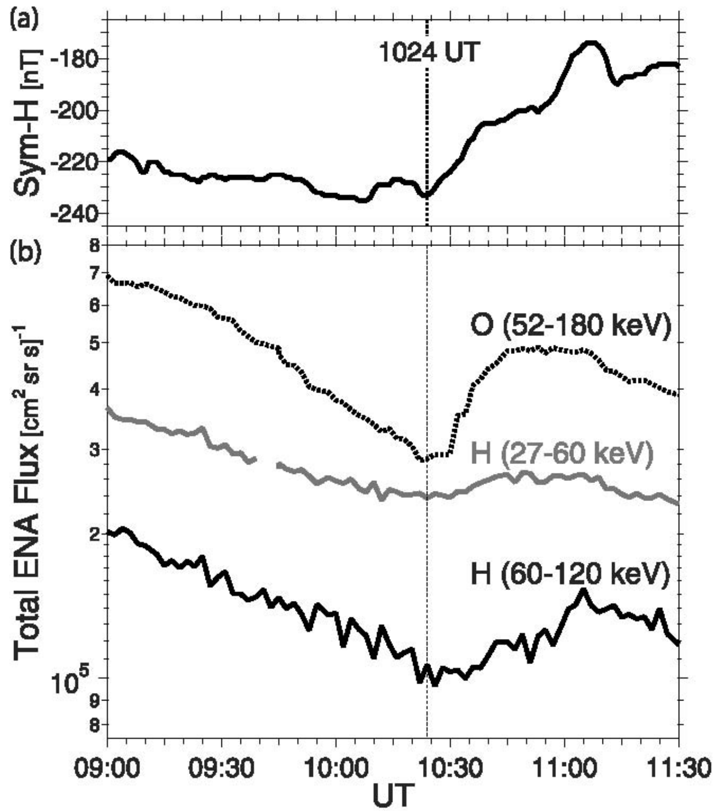
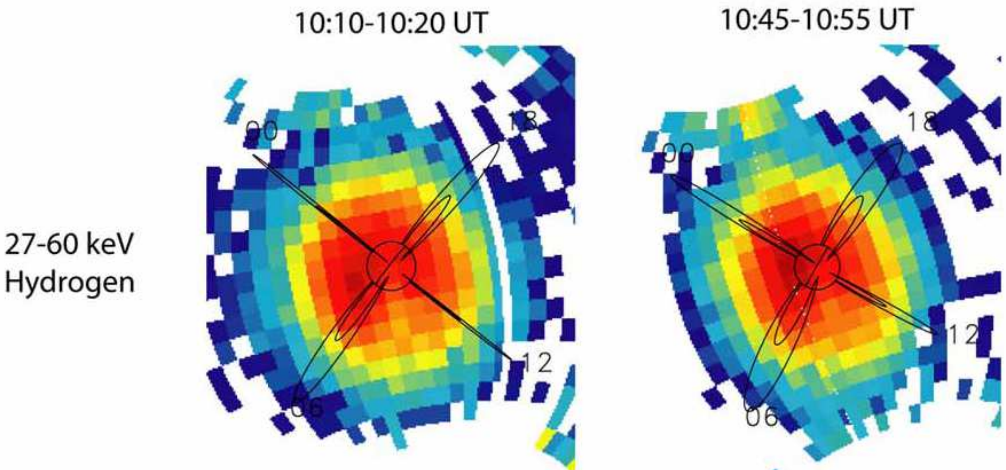


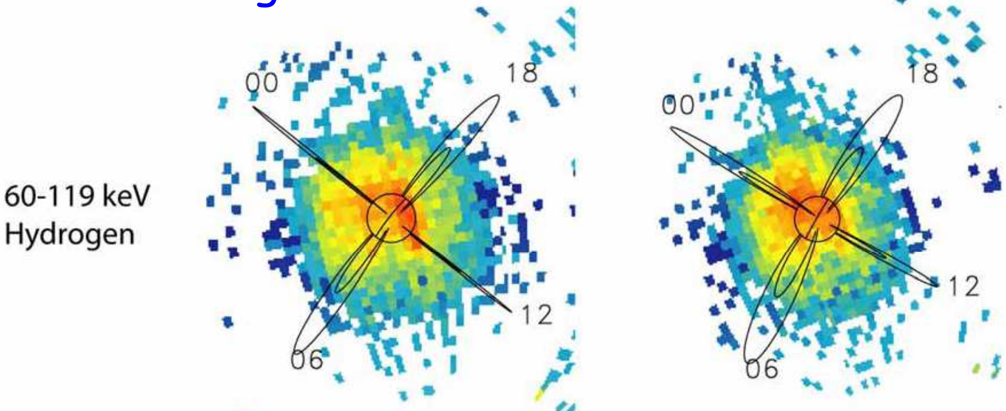




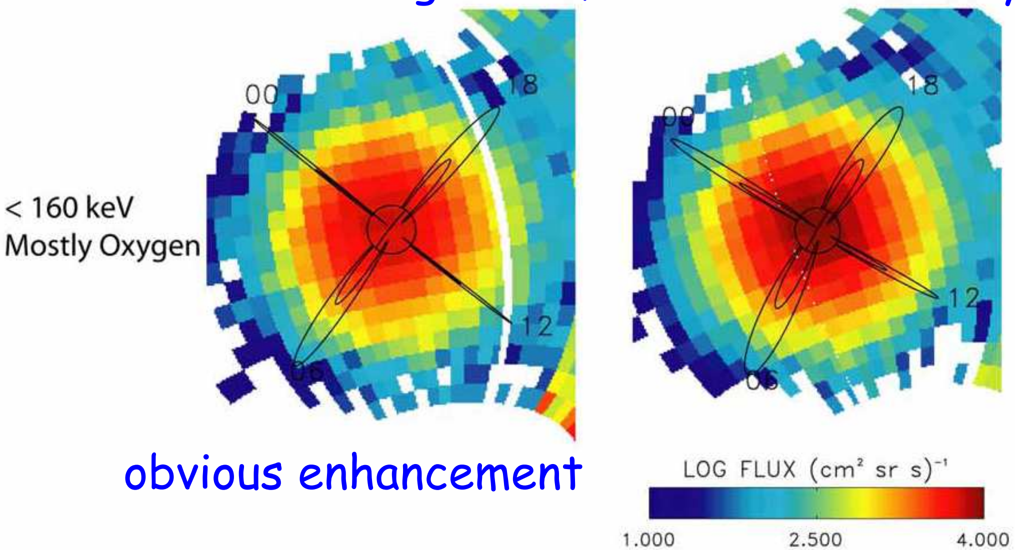
Ohtani et al. (2005)



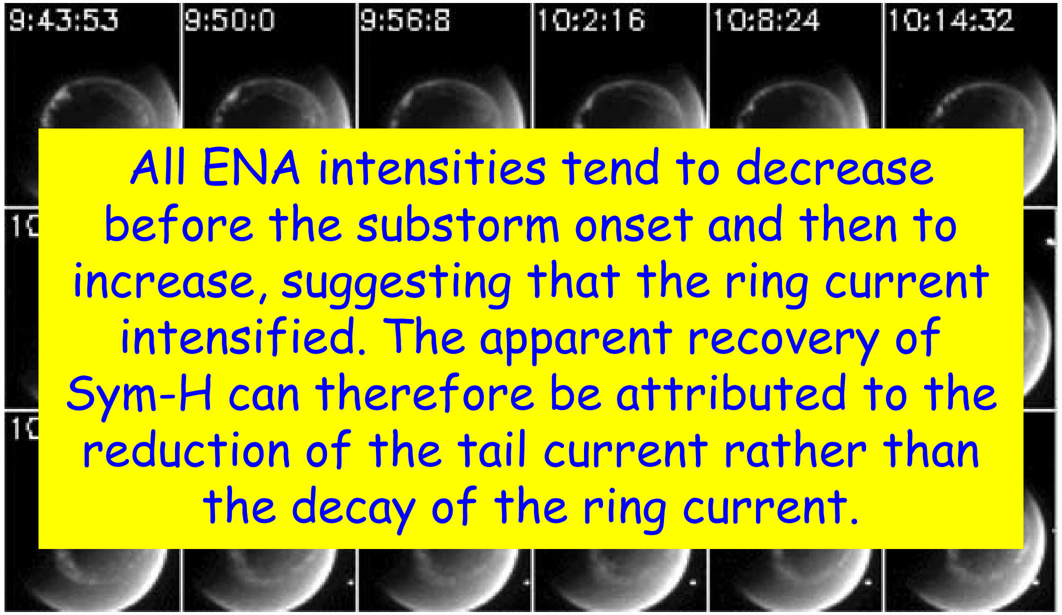
no significant difference



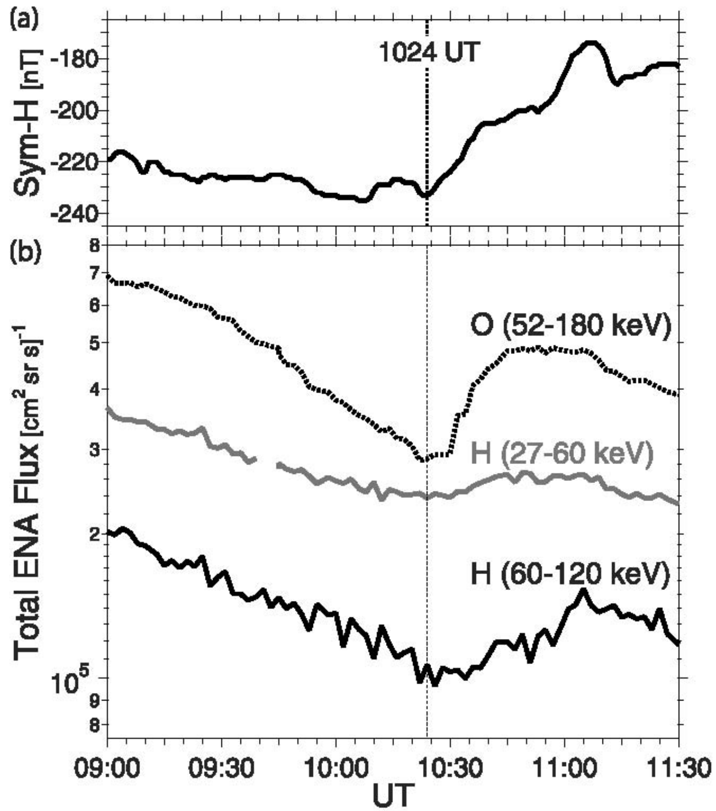
enhanced on the nightside, reduced on the dayside



obvious enhancement



All ENA intensities tend to decrease before the substorm onset and then to increase, suggesting that the ring current intensified. The apparent recovery of Sym-H can therefore be attributed to the reduction of the tail current rather than the decay of the ring current.



Ohtani et al. (2005)