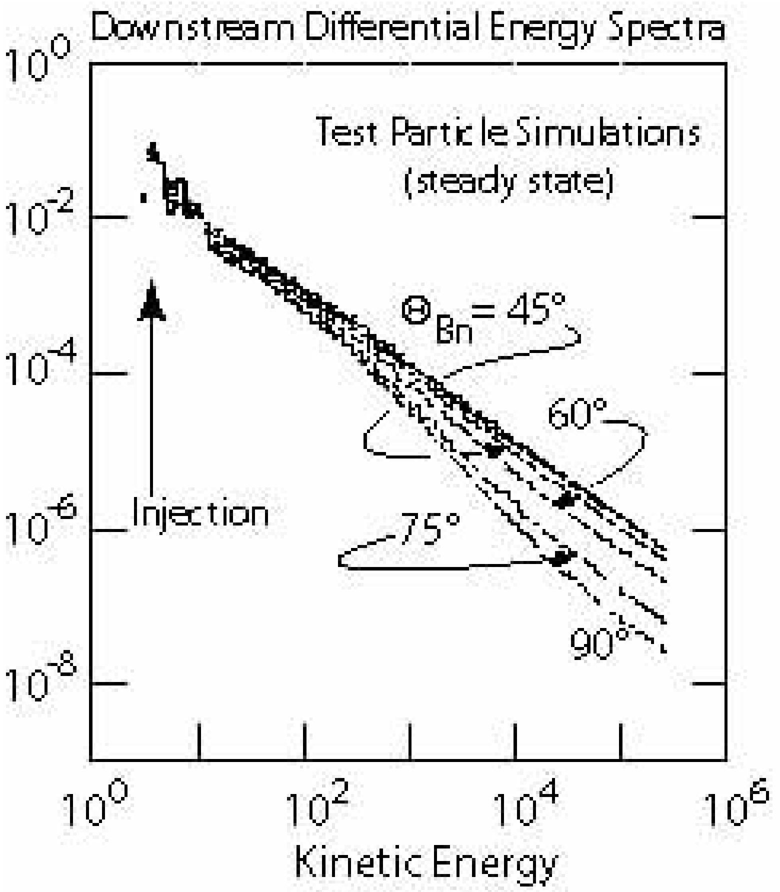
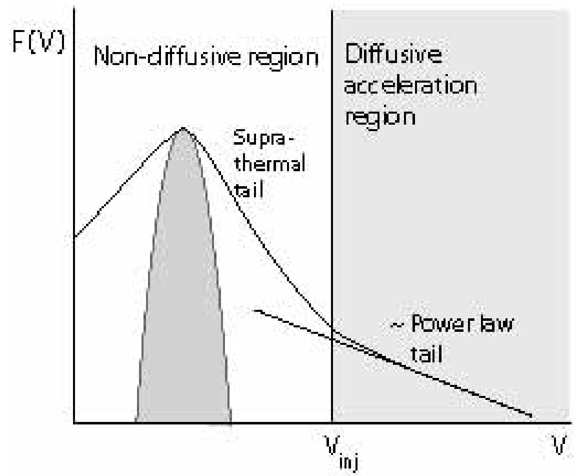
Test particle simulations of shock acceleration injecting protons into a pre-existing shock at different shock normal angles.





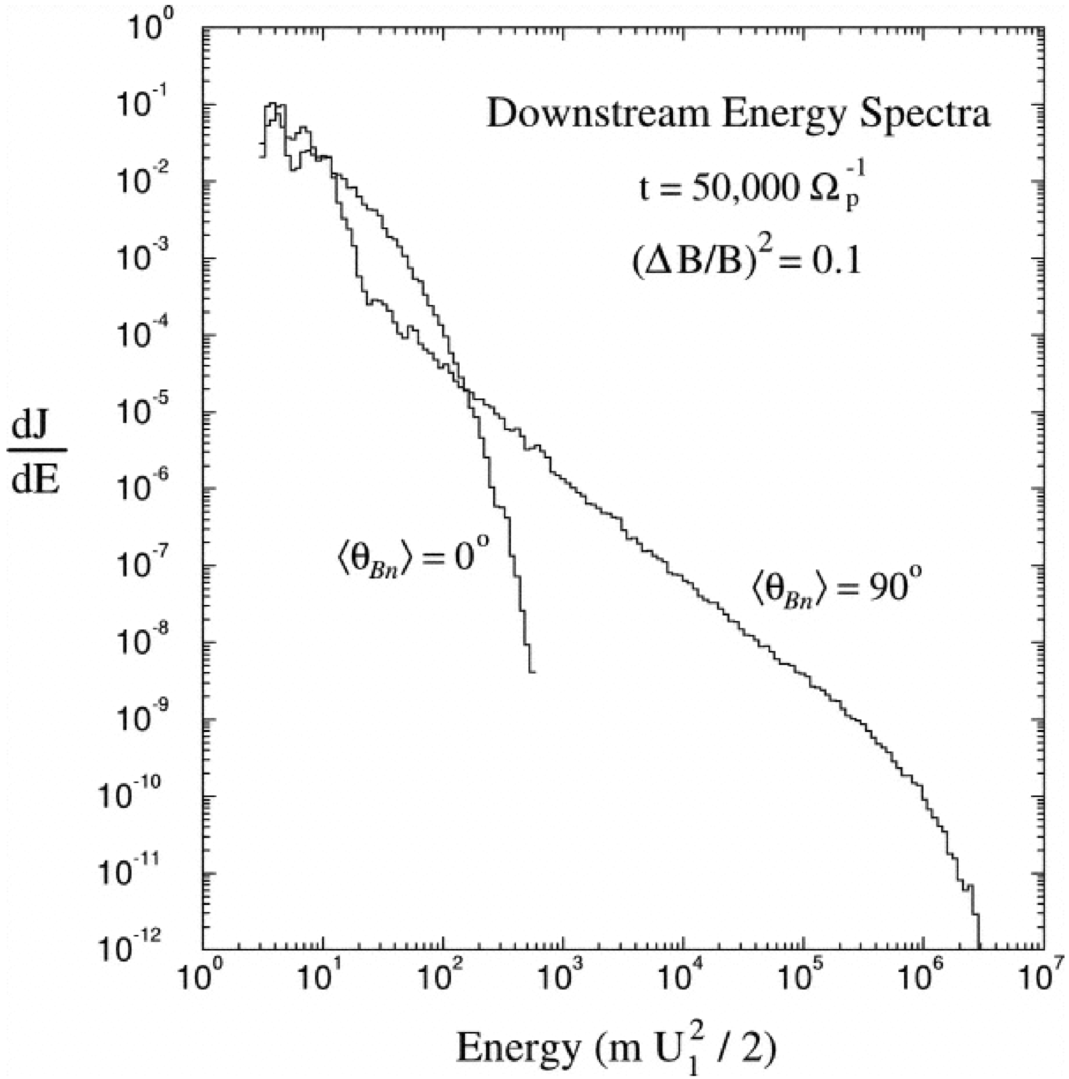
→Note that the flattest spectrum is obtained for small Θ_{Bn} in the quasi-parallel shock, but the dependence is weak.

→The difference is only in the flux with the quasi-parallel shock generated flux at given energy about one order of magnitude larger than the quasi-perpendicular shock generated flux.

→Nevertheless, time dependent test particle simulations of the same kind show that even though the fluxes are low the quasi-perpendicular shock accelerates particles to higher energy at a given time than the quasi-parallel shock.

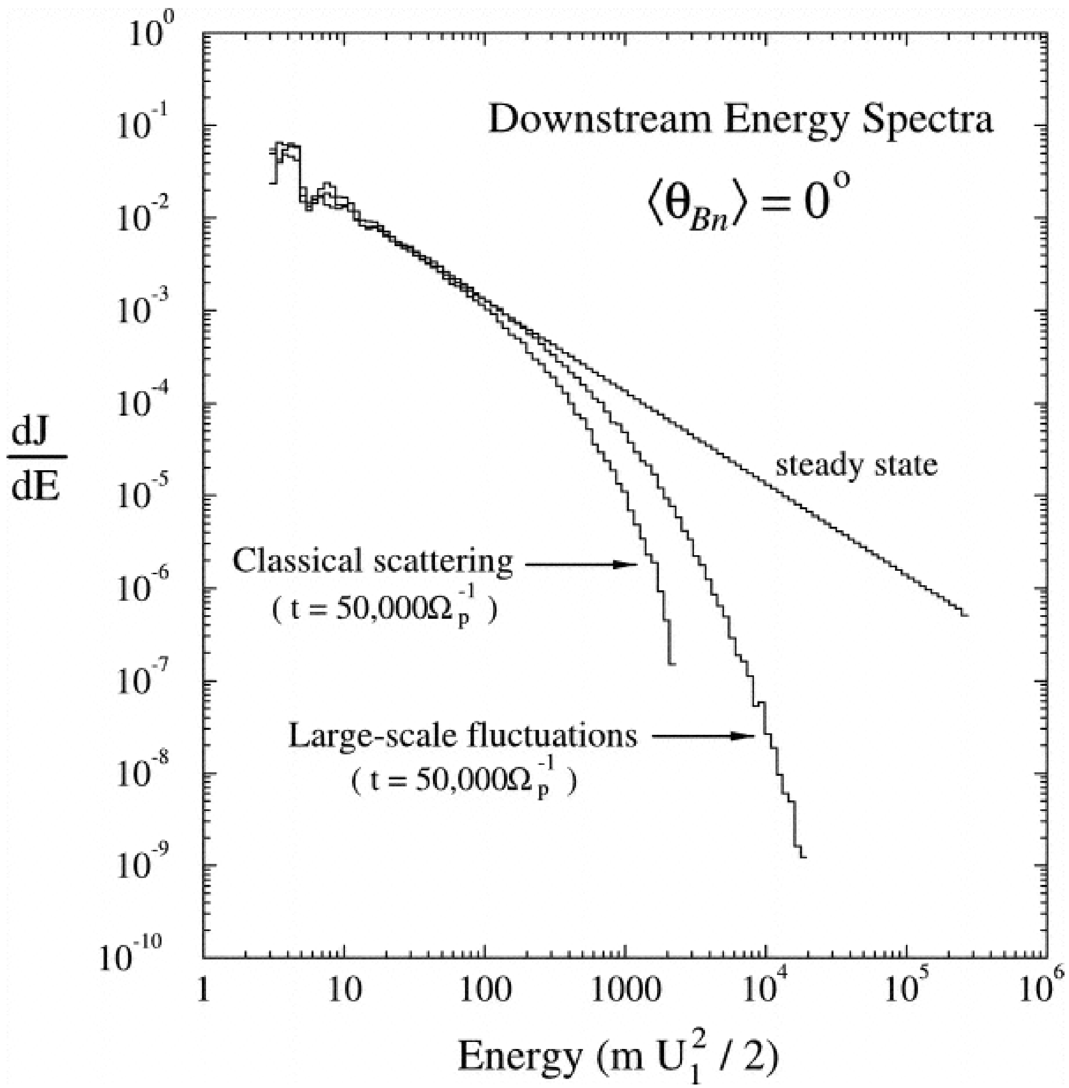
→In other words, quasi-perpendicular shocks show a higher acceleration rate than quasi-parallel shocks.

http://inspirehep.net/record/789082/files/chap6-fig-testsim.png



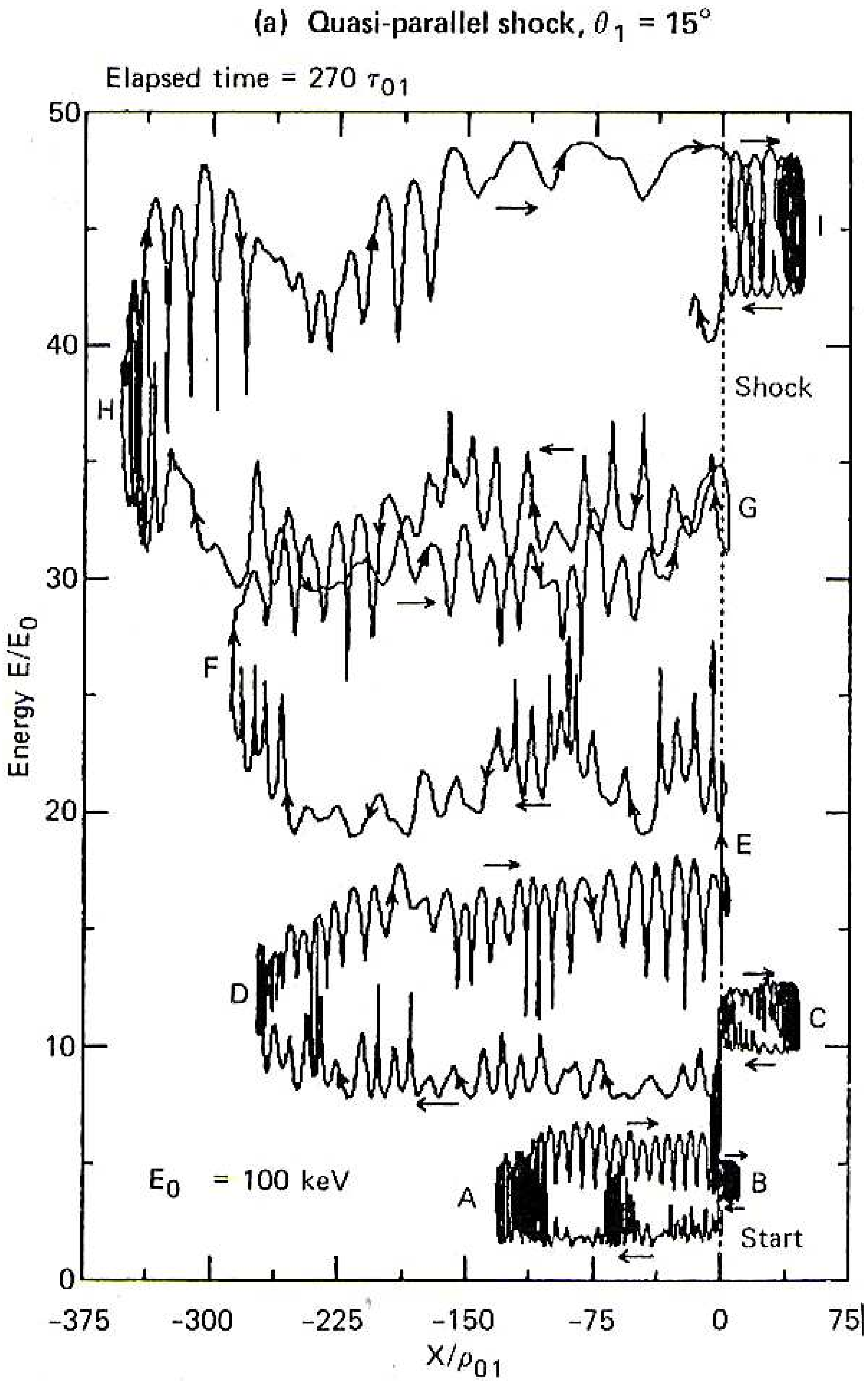
For the weak turbulence case, the acceleration rate at a parallel shock is very small because the particle mean free paths are very large. However, the perpendicular shock readily accelerates particles to very high energies because these particles are capable of diffusing normal to the mean magnetic field direction by following meandering magnetic field lines.

http://iopscience.iop.org/article/10.1086/429265/meta

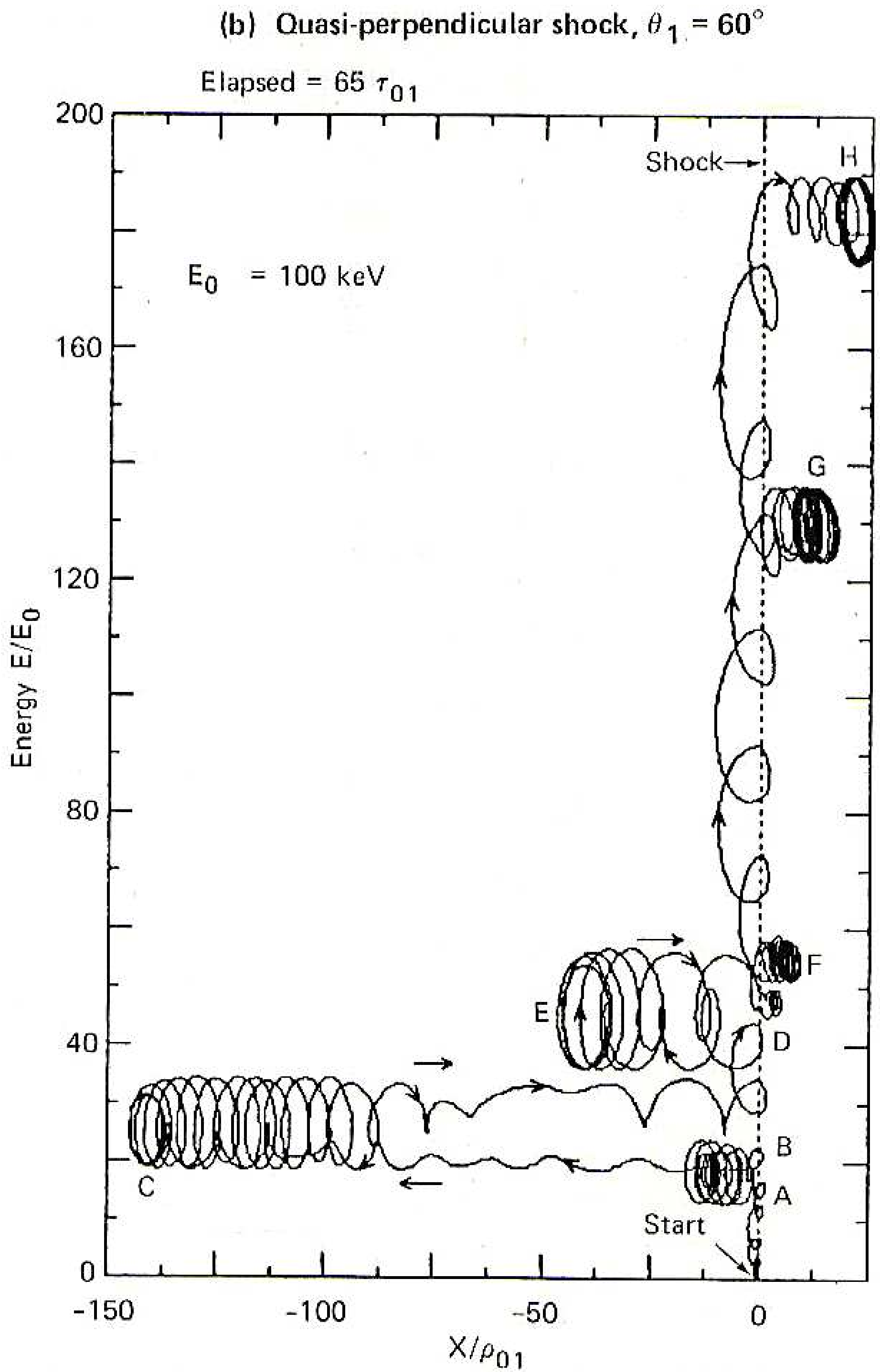


The large-scale fluctuations lead to a higher acceleration rate at a parallel shock compared to the case of simple first-order Fermi acceleration (the classical scattering case). This is due to the fact that, at times, the local magnetic field is more oblique to the shock normal. Thus, even for a parallel shock (on average), acceleration can occur locally at the shock because of drift acceleration.

http://iopscience.iop.org/article/10.1086/429265/meta

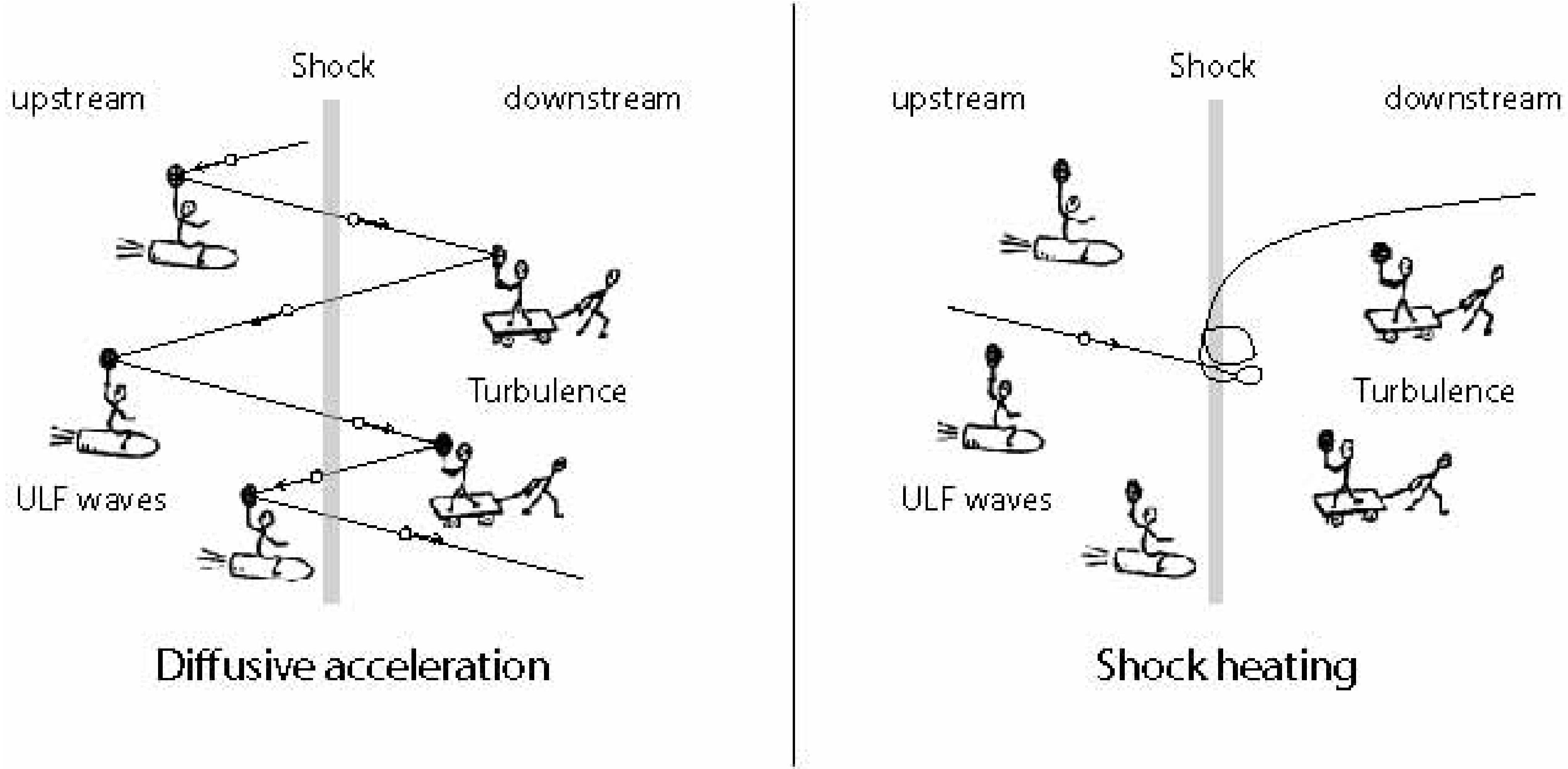


Slower Acceleration



More Rapid Acceleration

Decker (1988)



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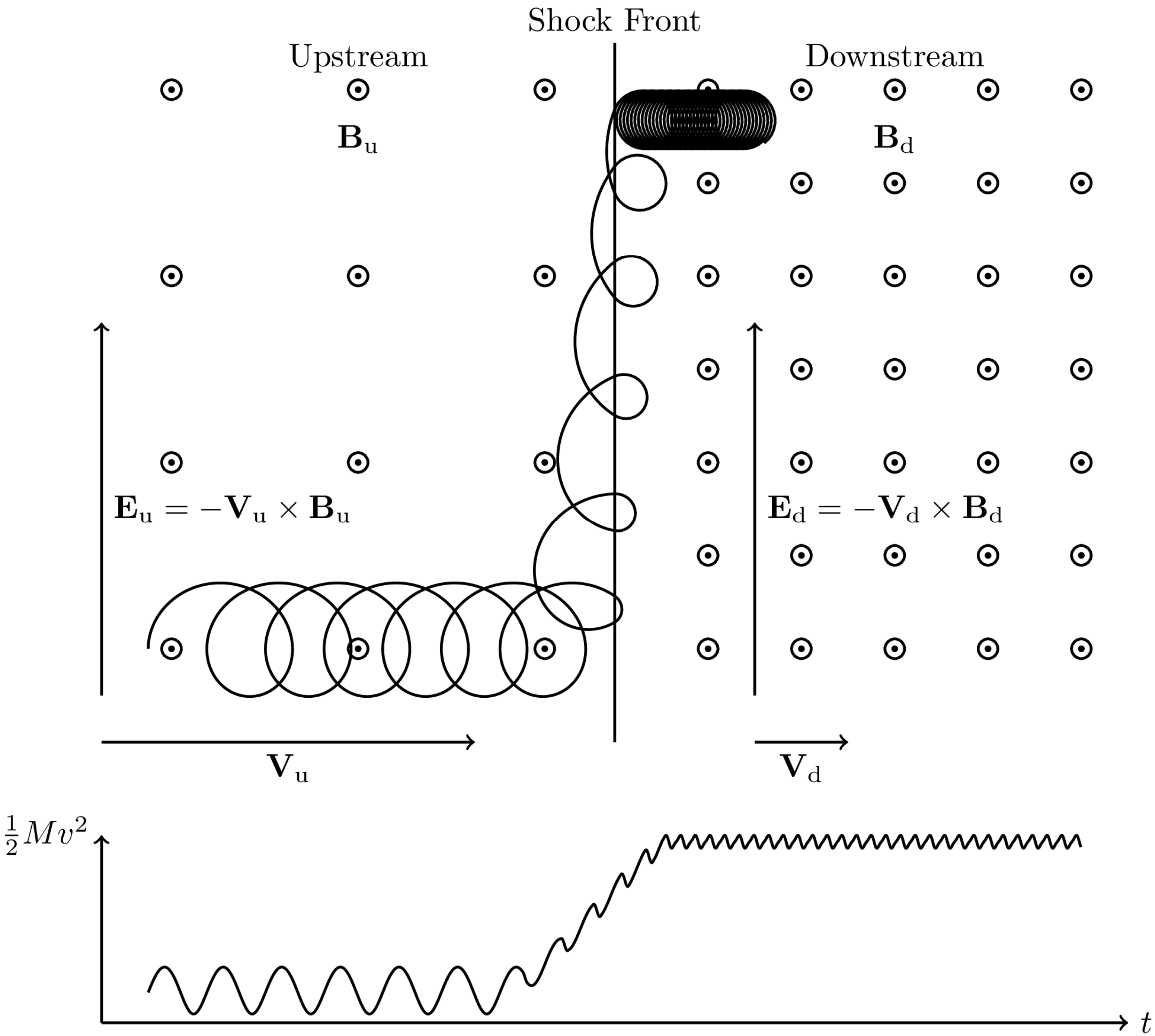
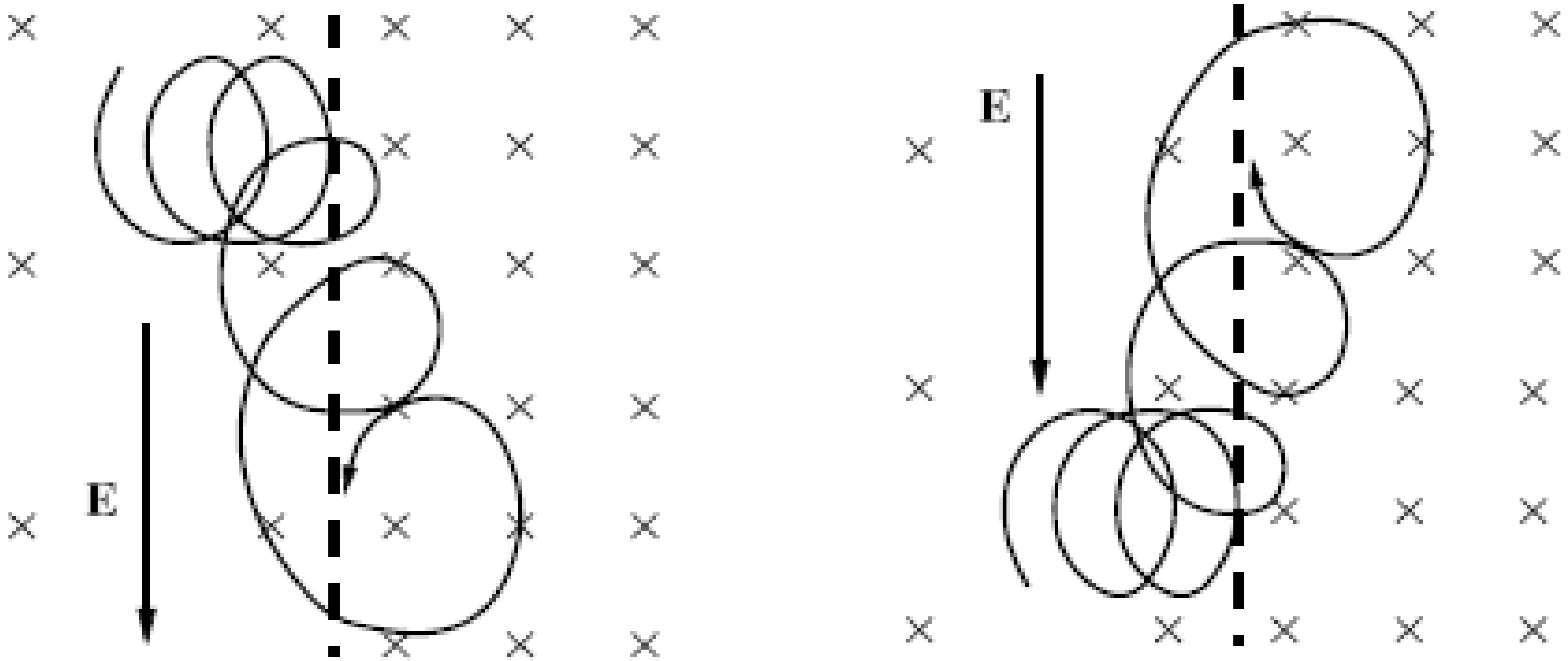
Shock Drift Acceleration



http://sprg.ssl.berkeley.edu/~pulupa/illustrations/shockdrift/shockdrift.png

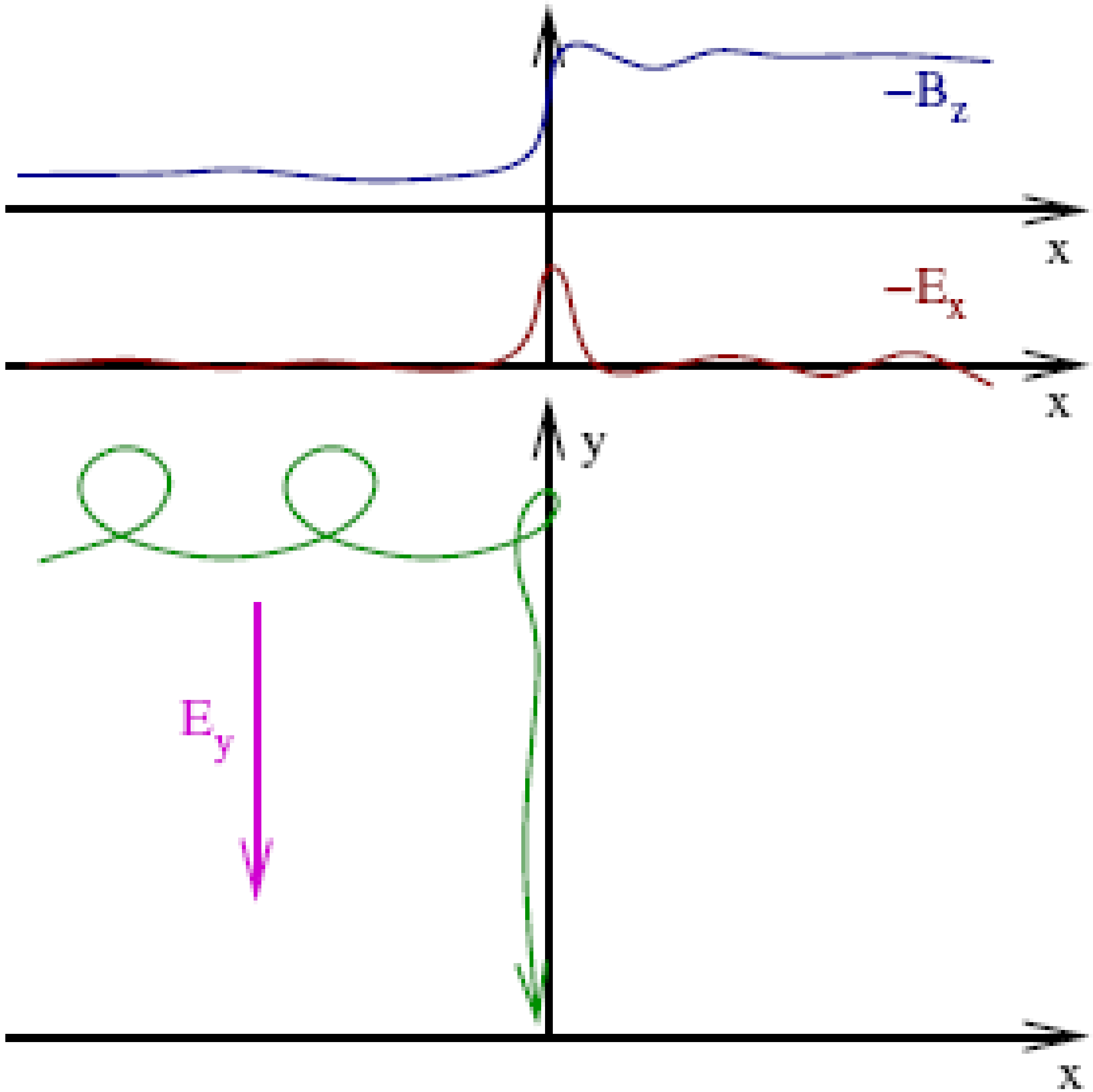


ions drift parallel to the E-field
electrons drift anti-parallel to the E-field
→ both ions and electrons are accelerated

Operates in oblique shocks as well.

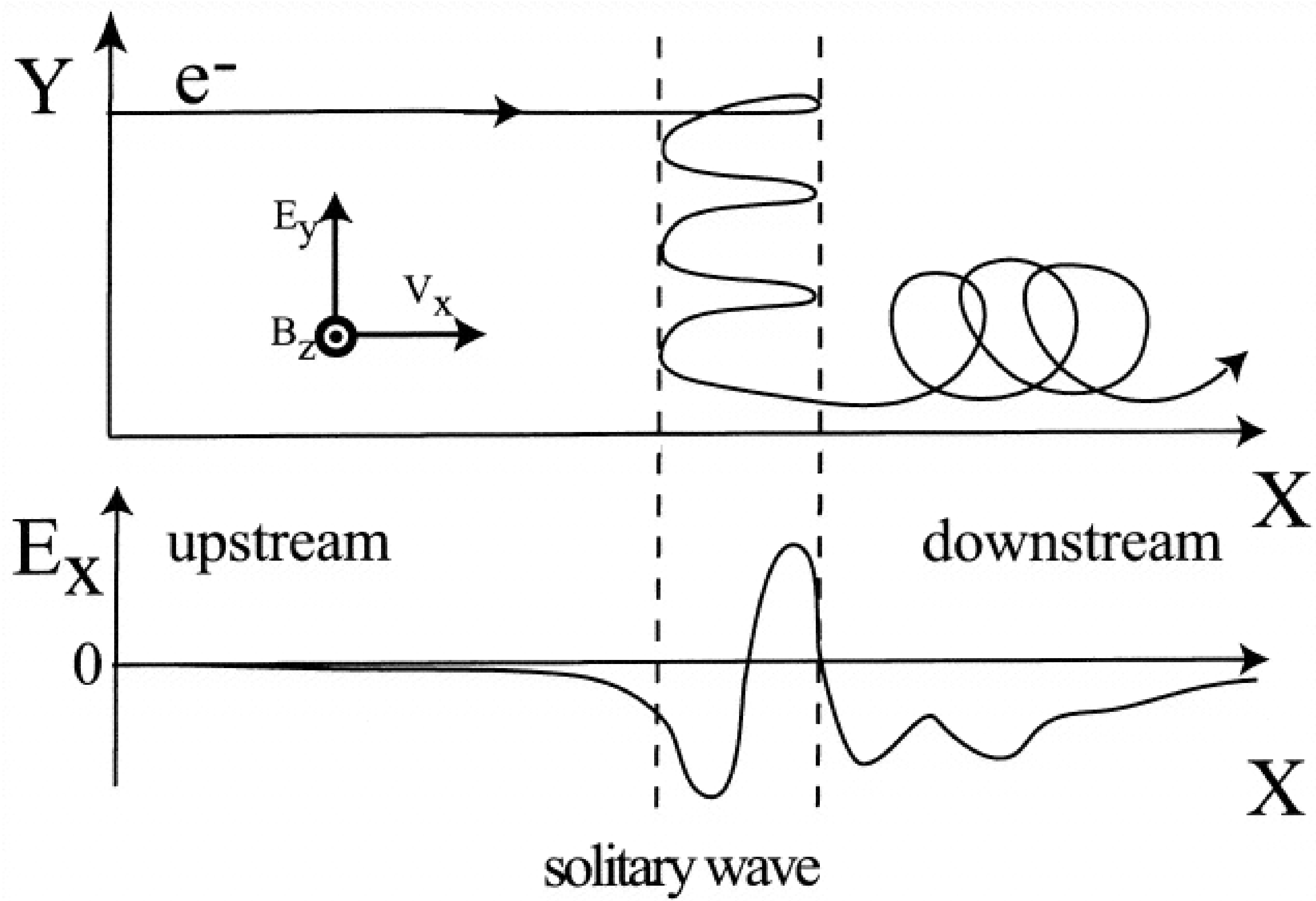
Vainio (2009)

Shock Surfing Acceleration



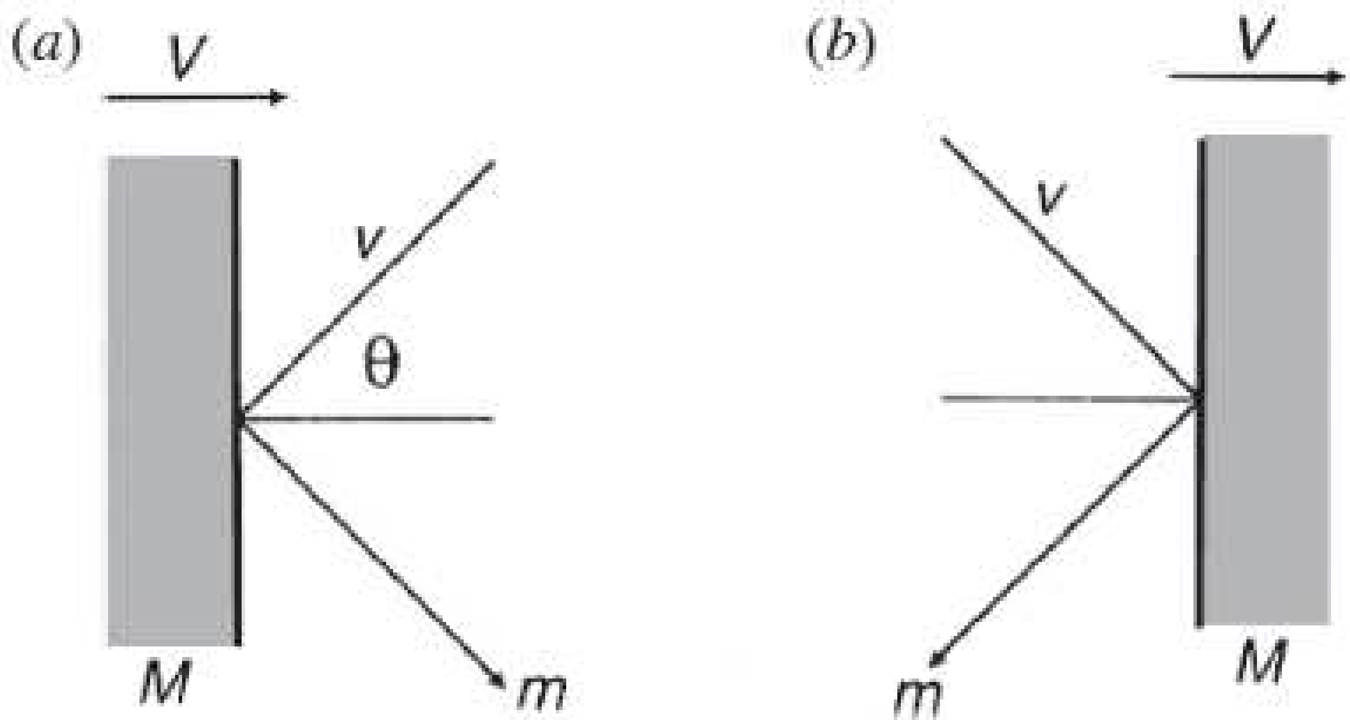
Surfing requires an almost exactly perpendicular shock.

Vainio (2009)



http://iopscience.iop.org/article/10.1086/340454/fulltext/55422.fg6.html

Second-order Fermi acceleration



Energy is (a) gained during a head-on collision and (b) lost during a trailing collision.

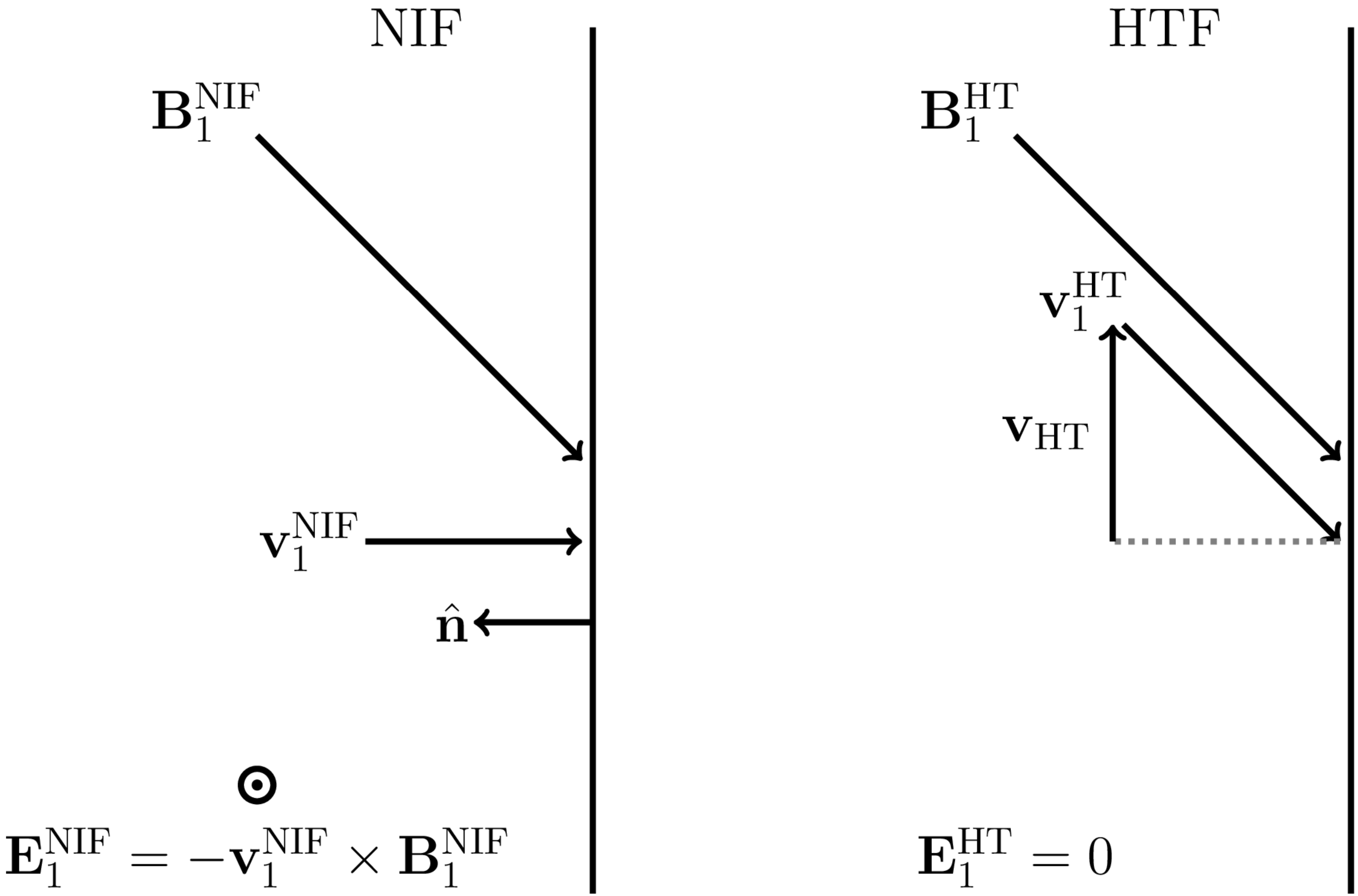
Cosmic rays bouncing between moving ISM clouds. Gain energy with head-on collision. Lose energy with overtaking collision.

This second-order process was not fast enough to overcome ionization losses for heavy elements in ISM.

https://www.cfa.harvard.edu/~namurphy/Lectures/Ay253_08_ParticleAccel.pdf

de Hoffmann-Teller Frame

(normal incidence frame)



http://sprg.ssl.berkeley.edu/~pulupa/illustrations/htframe/htframe.png

For $r_g \lll L$:

gyroradius

characteristic length

MHD still valid for bulk plasma.

SZPs can be treated as test particles

single particle motions in $\vec{E}$ & $\vec{B}$ fields

if $r_g \sim L$

1D diffusion eq:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial f}{\partial x}\right)$$

diffusion coefficient

$\propto v \lambda$

mean free path

particle's speed

usually:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left(k_\perp \frac{\partial f}{\partial x}\right) + \frac{\partial}{\partial z}\left(k_\parallel \frac{\partial f}{\partial z}\right)$$

In the heliosphere:

$$T_s \gg T_g$$ ← gyro period

charged particle to scatter

⇒ particles tend to move much more closely along $\vec{B}$ ⇒ $\kappa_\perp \ll \kappa_\parallel$

However, the perpendicular transport is the most important in many astrophysical plasmas of interest.

1D diffusion eq:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left(\kappa \frac{\partial f}{\partial x}\right)$$

diffusion coefficient

$\propto v \lambda$ ← mean free path

particle's speed

usually:

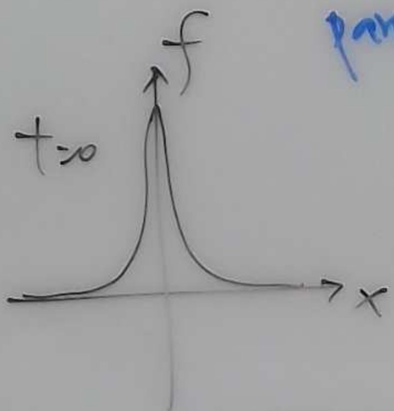
$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left(\kappa_\perp \frac{\partial f}{\partial x}\right) + \frac{\partial}{\partial z}\left(\kappa_\parallel \frac{\partial f}{\partial z}\right)$$

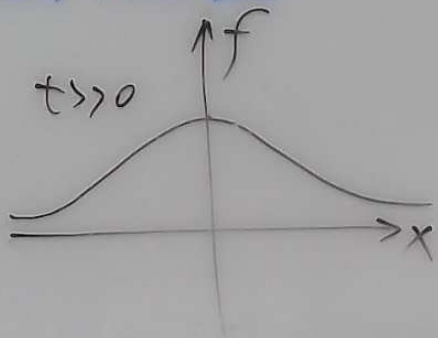
the solution of 1D diffusion eq. for an impulsive injection of particles at $x=0$, $t=0$:

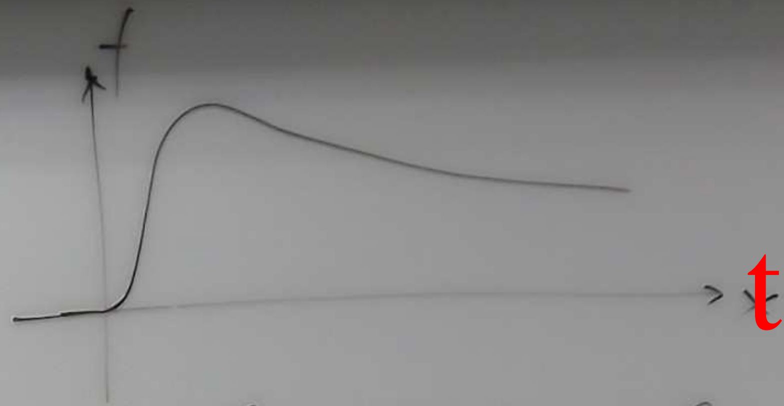
e.g. impulsive SEP

$$f(x,t) = \frac{N_0}{\sqrt{4\pi\kappa t}}\, e^{\left(-\frac{x^2}{4\kappa t}\right)}$$

number of particles released







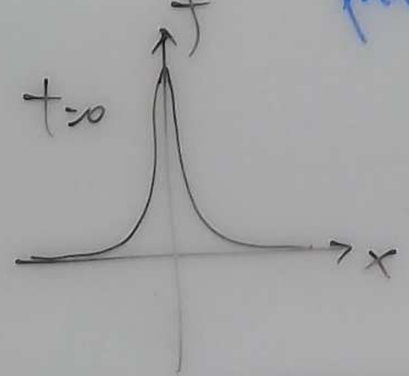
the earliest arriving particles suffer very little pitch-angle scattering, and therefore the transport eq. is not useful for describing these particles, but is adequate to describe the long-time behavior.

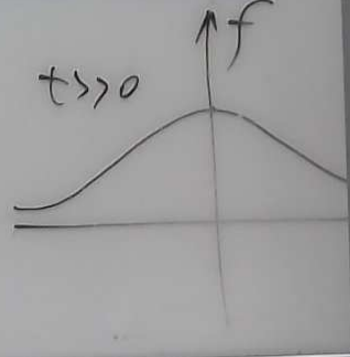
the solution of 1D diffusion eq. for an impulsive injection of particles at $x=0$, $t=0$:

e.g. impulsive SEP

$$f(x,t) = \frac{N_0}{\sqrt{4\pi \kappa t}}\, e^{\left(-\frac{x^2}{4\kappa t}\right)}$$

number of particles released





In the frame moving with flow, the eq. becomes "convection-diffusion" eq.

$$\frac{\partial f}{\partial t} = \underbrace{\frac{\partial}{\partial x}\left(\kappa \frac{\partial f}{\partial x}\right)}_{\text{diffusion}} - \underbrace{u\frac{\partial f}{\partial x}}_{\text{convection}}$$

the solution is

$$f(x,t) = \frac{N_0}{\sqrt{4\pi \kappa t}}\, e^{-\frac{(x-ut)^2}{4\kappa t}}$$

Including energy change, drifts, convection, diffusion, ...

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x_i}\left(\underbrace{\kappa_{ij}}_{\text{diffusion tensor}} \frac{\partial f}{\partial x_j}\right) - \vec{u}\cdot\nabla f + \frac{1}{3}(\nabla\cdot\vec{u})\underbrace{p}_{\text{momentum}}\frac{\partial f}{\partial p} \quad (+\text{sources} - \text{losses})$$

cosmic-ray transport eq.
Parker, 1965.

1D.

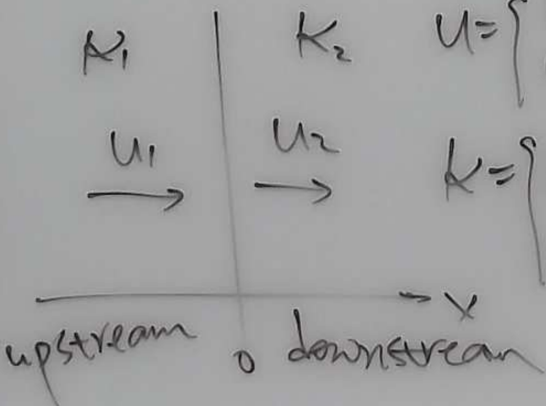
$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\kappa\frac{\partial f}{\partial x} - u\frac{\partial f}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial x}\underbrace{p\frac{\partial f}{\partial p}}_{=\frac{\partial f}{\partial \ln p}}$$

B.C.

$f(-\infty, p) \to 0$

$f(\infty, p) \to$ finite

$f_1(0, p) = f_2(0, p)$



$$u = \begin{cases} u_1, & x < 0 \\ u_2, & x \geq 0 \end{cases}$$

$$\kappa = \begin{cases} \kappa_1(p), & x < 0 \\ \kappa_2(p), & x \geq 0 \end{cases}$$

steady state $\Leftarrow$ i.e. $\frac{\partial}{\partial t} = 0$.

(i) upstream $(x < 0)$

$$0 = \frac{\partial}{\partial x}\kappa\frac{\partial f}{\partial x} - u\frac{\partial f}{\partial x} + \frac{1}{3}\cancel{\frac{\partial u}{\partial x}}\frac{\partial f}{\partial \ln p}$$

0 $(\because u = u_1)$

$$\Rightarrow 0 = \kappa_1\frac{d^2 f}{dx^2} - u_1\frac{df}{dx}$$

$$\Rightarrow f = \underbrace{A_1}_{\text{fun. of } p} e^{\frac{u_1}{\kappa_1}x} + \underbrace{B_1}_{\text{fun. of } p}$$

B.C. $f(-\infty, p) \to 0 \Rightarrow 0 = A_1 e^{-\infty} + B_1$

$\Rightarrow B_1 = 0$

$\Rightarrow f = A_1 e^{\frac{u_1}{\kappa_1}x}$

1D.

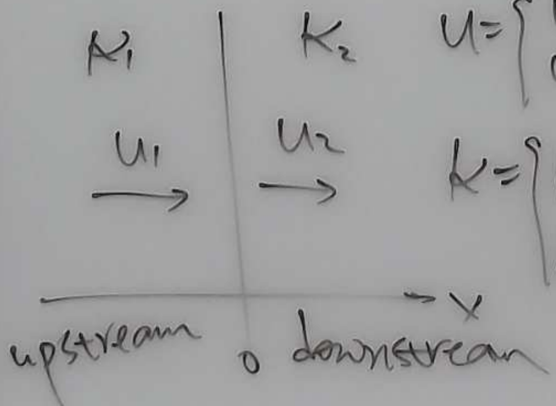
$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\kappa\frac{\partial f}{\partial x} - u\frac{\partial f}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial x}\underbrace{p\frac{\partial f}{\partial p}}_{=\frac{\partial f}{\partial \ln p}}$$

B.C.

$f(-\infty, p) \to 0$

$f(\infty, p) \to \text{finite}$

$f_1(0, p) = f_2(0, p)$



$$u = \begin{cases} u_1, & x<0 \\ u_2, & x \geq 0 \end{cases}$$

$$\kappa = \begin{cases} \kappa_1(p), & x<0 \\ \kappa_2(p), & x \geq 0 \end{cases}$$

(ii) downstream $(x > 0)$

$$0 = \kappa_2 \frac{d^2 f}{dx^2} - u_2 \frac{df}{dx} \Rightarrow f = A_2 e^{\frac{u_2}{\kappa_2}x} + B_2$$

B.C. $f(\infty, p) \to \text{finite} \Rightarrow \text{finite} = A_2 e^{\infty} + B_2$

$\Rightarrow A_2 = 0$

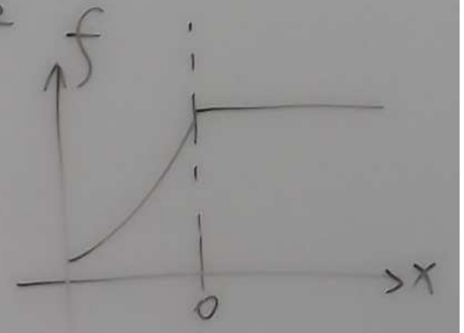
$\Rightarrow f = B_2$

moreover, $f_1(0, p) = f_2(0, p)$

$\Rightarrow f_1 = A_1 e^{\frac{u_1}{\kappa_1}(0)} = B_2 = f_2$

$\Rightarrow \quad A_1 = B_2 \equiv A$

$$\therefore f(x, p) = \begin{cases} A(p)\, e^{\frac{u_1}{\kappa_1}x}, & x<0 \\ A(p), & x \geq 0 \end{cases}$$



$A(\varphi) = ?$

derived from $\displaystyle\int_{-\epsilon}^{\epsilon} \frac{\partial f}{\partial t}\,dx$, for $\epsilon \ll 1$

$$\Rightarrow \int_{-\epsilon}^{\epsilon} \frac{\partial}{\partial x}\left(k \frac{\partial f}{\partial x}\right) dx - \int_{-\epsilon}^{\epsilon} u \frac{\partial f}{\partial x}\,dx + \int_{-\epsilon}^{\epsilon} \frac{1}{3}\frac{\partial u}{\partial x}\frac{\partial f}{\partial \ln p}\,dx = 0$$

① ② ③

①: $\displaystyle\int_{-\epsilon}^{\epsilon} \frac{\partial}{\partial x}\left(k \frac{\partial f}{\partial x}\right) dx$

$$= k \frac{\partial f}{\partial x}\Big|_{-\epsilon}^{\epsilon}$$

$$\because f(x,p) = \begin{cases} A e^{\frac{u_1}{k_1}x}, & x<0 \\ A, & x \geq 0 \end{cases}$$

$$\Rightarrow \frac{\partial f}{\partial x} = \begin{cases} \frac{u_1}{k_1} A e^{\frac{u_1}{k_1}x}, & x<0 \\ 0, & x \geq 0 \end{cases}$$

$$= k_2 0 - k_1 \frac{u_1}{k_1} A e^{\frac{u_1}{k_1}\epsilon}$$

$$= -u_1 A e^{\frac{u_1}{k_1}\epsilon}$$

$$\simeq -u_1 A$$

② $-\int_{-\epsilon}^{\epsilon} u \frac{\partial f}{\partial x} dx$

$$= -\left(\int_{-\epsilon}^{0} u \frac{\partial f}{\partial x} dx + \int_{0}^{\epsilon} u \frac{\partial f}{\partial x} dx\right)$$

$$= -\left(u \int_{-\epsilon}^{0} \frac{\partial f}{\partial x} dx + u \int_{0}^{\epsilon} \frac{\partial f}{\partial x} dx\right)$$

$$= -\left(u_1 f \Big|_{-\epsilon}^{0} + u_2 f \Big|_{0}^{\epsilon}\right)$$

$$= -\left(u_1 \left(A e^{-\frac{u_1}{k_1}(0)} - A e^{-\frac{u_1}{k_1}\epsilon}\right) + u_2 (A - A)\right)$$

$$= -u_1 \left(A - A e^{-\frac{u_1}{k_1}\epsilon}\right) \simeq 0$$