Applications of Maxwell-Boltzmann Distribution (2) 量測恆星溫度

呂凌霄
Ling-Hsiao Lyu
Institute of Space Science, NCU

Classical Gas: Maxwell-Boltzmann Distribution

$$f(v_x, v_y, v_z) = \frac{n}{(\sqrt{2\pi})^3 \sigma^3} \exp\left[-\frac{v_x^2 + v_y^2 + v_z^2}{2\sigma^2}\right]$$

$$= \frac{n}{(\sqrt{2\pi k_B T / m})^3} \exp\left[-\frac{mv^2}{2k_B T}\right]$$

$$f(E_k) = \frac{n}{(\sqrt{2\pi k_B T / m})^3} \exp\left(-\frac{E_k}{k_B T}\right)$$

Classical Gas: Maxwell-Boltzmann Distribution
推廣到光與低溫物理

- Classical Gas: Maxwell-Boltzmann Distribution

$$f(E_k) = \frac{n}{(\sqrt{2\pi k_B T / m})^3} \exp(-\frac{E_k}{k_B T}) = \frac{f_0}{e^{E_k / k_B T}}$$

- Bosons: Bose-Einstein Distribution

$$f(E_k) = \frac{f_0}{e^{E_k / k_B T} - 1}$$

- Fermions: Fermi-Dirac Distribution

$$f(E_k) = \frac{f_0}{e^{(E_k - \mu)/k_B T} + 1}$$

where μ denotes certain kind of potential energy

玻色子的spin為整數，可分為基本玻色子與組合玻色子(如:原子)
其中基本玻色子(Elementary bosons)為傳遞作用力的粒子，如：
光子(photon)傳遞電磁力
膠子(gluon)傳遞強作用力
W,Z玻色子傳遞弱作用力
重子(graviton)傳遞重力~~（尚未發現）~~
也就是 Higgs ~~(我不懂)（尚未發現）~~
(最近被發現了，今年2013年的諾貝爾獎)
玻色子同性相吸（gays!）

$$f(E_k) = \frac{f_0}{e^{E_k/k_B T} - 1}$$

- Fermions: Fermi-Dirac Distribution

$$f(E_k) = \frac{f_0}{e^{(E_k - \mu)/k_B T} + 1}$$

where μ denotes certain kind of potential energy

費米子的spin為半整數，也可分為基本費米子(elementary fermions)如：
電子(electron)
微中子(neutrinos)
夸克子(quarks)
與組合費米子(Composite fermions)如：
質子(proton)
中子(neutron)
費米子是組成物質的元素。
費米子同性相斥(guys!) 所以溫度越低，密度應該越少，除非有很大的束縛力(如重力)，才能製造低溫高密度的費米子簡併態。

- Fermions: Fermi-Dirac Distribution

$$f(E_k) = \frac{f_0}{e^{(E_k - \mu)/k_B T} + 1}$$

where μ denotes certain kind of potential energy

Classical Gas: Maxwell-Boltzmann Distribution
推廣到光與低溫物理

- Classical Gas: Maxwell-Boltzmann Distribution

$$f(E_k) = \frac{f_0}{e^{(E_k - \mu)/k_B T}}$$

- Bosons: Bose-Einstein Distribution

$$f(E_k) = \frac{f_0}{e^{(E_k - \mu)/k_B T} - 1}$$

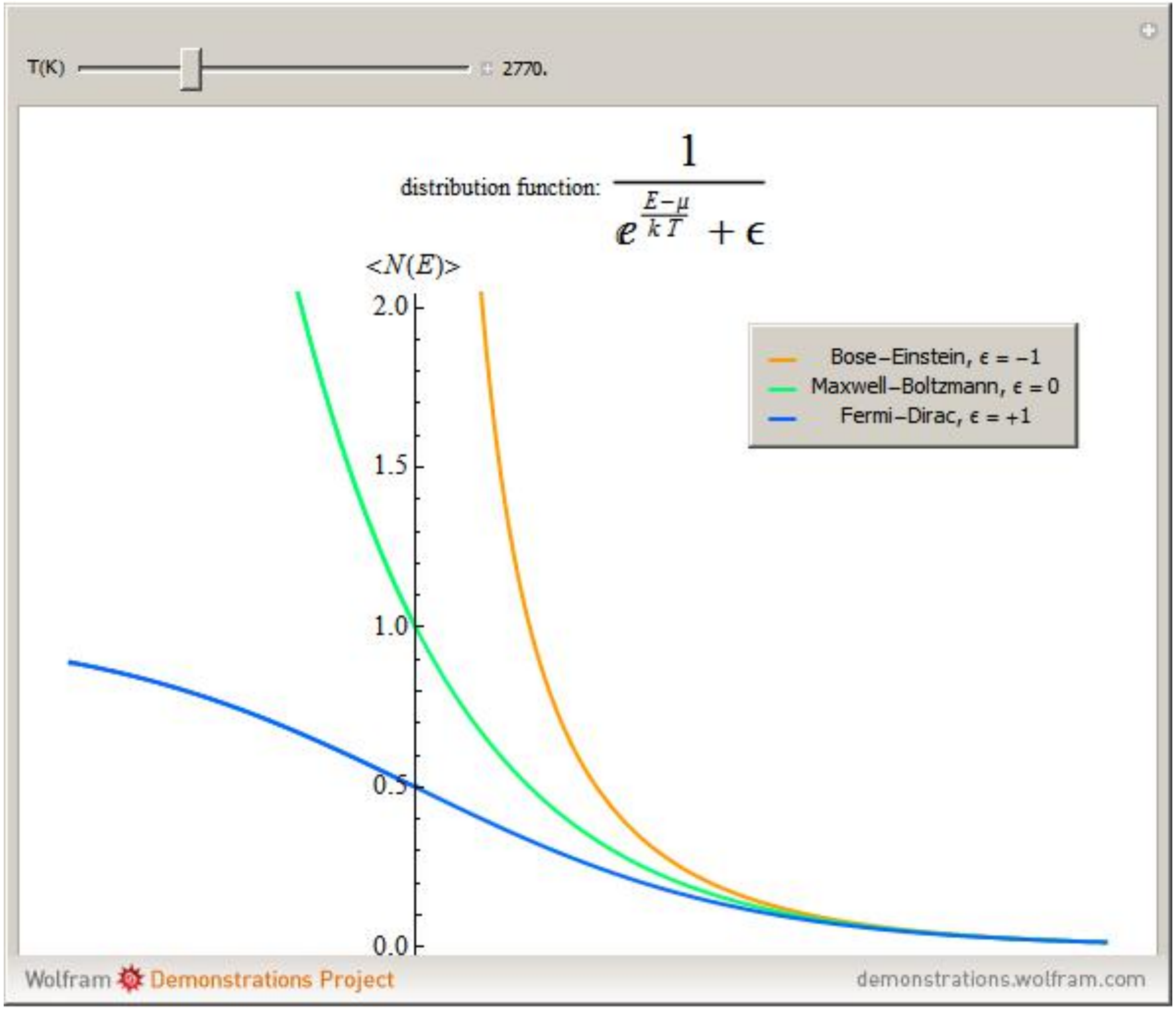
- Fermions: Fermi-Dirac Distribution

$$f(E_k) = \frac{f_0}{e^{(E_k - \mu)/k_B T} + 1}$$

where μ denotes certain kind of potential energy

http://demonstrations.wolfram.com/BoseEinsteinFermiDiracAndMaxwellBoltzmannStatistics/

Also, see
課本269頁



Applications of Maxwell-Boltzmann Distribution 推廣到黑體輻射

- Classical Gas: Maxwell-Boltzmann Distribution

$$f(E_k) = \frac{f_0}{e^{E_k/k_BT}}$$

- Bosons: Bose-Einstein Distribution

$$f(\varepsilon) = \frac{f_0}{e^{\varepsilon/k_BT} - 1} \quad \text{where } \varepsilon = h\nu = h\frac{c}{\lambda} = h\frac{cn}{2L} \text{ and } n = \sqrt{n_x^2 + n_y^2 + n_z^2}$$

- Summing over Modes (for both left-hand and right-hand polarized waves). Let $f_0 = 1$. It yields

$$U = 2\sum_{n_x}\sum_{n_y}\sum_{n_z}\varepsilon f(\varepsilon) = \sum_{n_x}\sum_{n_y}\sum_{n_z}\frac{hcn}{L}\frac{1}{e^{hcn/2Lk_BT} - 1}$$

Applications of Maxwell-Boltzmann Distribution 推廣到黑體輻射

- Summing over Modes

$$U = \int_0^\infty dn_x \int_0^\infty dn_y \int_0^\infty dn_z \left(\frac{hcn}{L} \frac{1}{e^{hcn/2Lk_BT} - 1}\right)$$

$$= \int_0^\infty dn \int_0^{\pi/2} n\, d\theta \int_0^{\pi/2} n \sin\theta\, d\phi \frac{hcn}{L} \frac{1}{e^{hcn/2Lk_BT} - 1}$$

$$= \int_0^\infty dn \frac{4\pi n^2}{8} \frac{hcn}{L} \frac{1}{e^{hcn/2Lk_BT} - 1}$$

For $\varepsilon = h\dfrac{cn}{2L}$ it yields $d\varepsilon = \dfrac{hc}{2L} dn$ and

$$U = L^3 \int_0^\infty d\varepsilon \frac{8\pi\varepsilon^3 / (hc)^3}{e^{\varepsilon/k_BT} - 1}$$

Applications of Maxwell-Boltzmann Distribution 推廣到黑體輻射

- Summing over modes

$$U = L^3 \int_0^\infty d\varepsilon \frac{8\pi\varepsilon^3 / (hc)^3}{e^{\varepsilon / k_B T} - 1}$$

- Let the energy density to be $u(\varepsilon)$, such that total energy per unit volume ($V = L^3$) becomes

$$\frac{U}{V} = \frac{U}{L^3} = \int_0^\infty d\varepsilon \frac{8\pi\varepsilon^3 / (hc)^3}{e^{\varepsilon / k_B T} - 1} = \int_0^\infty u(\varepsilon) d\varepsilon$$

- Thus, we have obtain the Planck Spectrum (the energy density per unit photon energy), i.e.,

$$u(\varepsilon) = \frac{8\pi\varepsilon^3 / (hc)^3}{e^{\varepsilon / k_B T} - 1}$$

Applications of Maxwell-Boltzmann Distribution 推廣到黑體輻射

- Let $x = \varepsilon / k_B T$ The total energy per unit volume becomes

$$\frac{U}{V} = \int_0^\infty d\varepsilon \frac{8\pi\varepsilon^3/(hc)^3}{e^{\varepsilon/k_B T} - 1} = \int_0^\infty k_B T\, dx \frac{8\pi x^3 (k_B T/hc)^3}{e^x - 1}$$

$$= \frac{8\pi (k_B T)^4}{(hc)^3} \int_0^\infty dx \frac{x^3}{e^x - 1}$$

Since $\int_0^\infty dx \frac{x^3}{e^x - 1} = \frac{\pi^4}{15}$ (不必記，因為我也背不下來！)

It yields $\frac{U}{V} = \frac{8\pi^5 (k_B T)^4}{15(hc)^3}$

The Stefan-Boltzmann constant

$$\sigma = \frac{2\pi^5 k_B^4}{15 h^3 c^2} = 5.67 \times 10^{-8} \frac{\mathrm{W}}{\mathrm{m^2 K^4}}$$

- Blackbody Radiation: power per unit area $\frac{c}{4}\frac{U}{V} = \frac{2\pi^5 (k_B T)^4}{15 h^3 c^2} = \sigma T^4$ (see pp.301-302)

Planck Spectrum

(e.g., Figures 7.19 & 7.20 on pages 294-296)

- The energy density per unit photon energy

$$u(\varepsilon) = \frac{8\pi\varepsilon^3 / (hc)^3}{e^{\varepsilon / k_B T} - 1}$$

Home Work 3.1: Plot the Planck spectrum $u(\varepsilon)$ at different temperature (e.g., T=1000ºK, 5000ºK, 10000ºK)

- The the energy density of the light per unit frequency

$$u(\nu) = \frac{8\pi\nu^3 / c^3}{e^{h\nu / k_B T} - 1} \; h$$

Home Work 3.2: Plot the Planck spectrum $u(\nu)$ at different temperature (e.g., T=1000ºK, 5000ºK, 10000ºK)

- The the energy density of the light per unit wavelength

$$u(\lambda) = \frac{8\pi / \lambda^3}{e^{hc / \lambda k_B T} - 1} \; \frac{hc}{\lambda^2}$$

Home Work 3.3: Plot the Planck spectrum $u(\lambda)$ at different temperature (e.g., T=1000ºK, 5000ºK, 10000ºK)

Applications of Blackbody Radiation & Planck Spectrum

- Scientists use the blackbody radiation or the Planck spectrum to determine the "temperature" of a distant star.
- The blackbody radiation model is not applicable to a diluted medium
- For a diluted medium, the spectra usually show discrete emission lines from the medium. (example: aurora)

Outlines of Thermal Dynamics

本學期課綱 黑字部份，已經教完了；藍字部份，待教

- Temperature and thermodynamic equilibrium 溫度與熱力平衡
- The ideal gas 理想氣體
- The distribution function in the velocity space 速度空間的分布函數
- Maxwellian distribution 馬克斯威爾分布函數
- Boltzmann equation 波茲曼方程式
- Energy equation 能量方程式
- Heat and Work 熱與功
- Enthalpy and Entropy 焓與熵
- Reversible and Irreversible Processes 可逆與不可逆過程
- Adiabatic process 絕熱過程
- Equation of state 狀態方程式
- Heat convection and heat conduction 熱對流與熱傳導
- The three laws in thermal dynamics 簡介熱力學的三個基本定律
- P-V-T 圖與相之轉移
- Bose-Einstein, Fermi-Dirac, and Maxwell-Boltzmann Statistics
- Blackbody Radiation and Planck Spectrum