

作業目的：幫助了解「連續的 (continuous) 機率密度函數 (probability density function) $f(x)$ 」與「離散的 (Discrete) 機率 (probability) $F_i = F(x_i)$ 」之間的關係。

Case 1.

Let $f(x)$ be a uniform probability density function in the domain $0 \leq x \leq 6$ and $f(x) = 0$ otherwise.

(a) Please write down the function form of the $f(x)$

$$f(x) = \begin{cases} _ & \text{if } -\infty < x < 0 \\ _ & \text{if } 0 \leq x \leq 6 \\ _ & \text{if } 6 < x < +\infty \end{cases}$$

(b) If the distribution of variables x is given by the probability density function $f(x)$, please determine the mean and the variances of the variables x .

Case 2.

Let the probability to have a result $x = x_i$ is equal to $F_i = F(x_i)$. Let us consider a uniform probability for all $x = x_i$.

(a) Please determine the probability $F_i = F(x_i)$, the mean, and the variance of the variables $x = x_i$, if $i = 1, 2, 3, 4, 5, 6$ and $x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 4, x_5 = 5, x_6 = 6$. (i.e., $x_i = i$)

(b) Please determine the probability $F_i = F(x_i)$, the mean, and the variance of the variables $x = x_i$, if $i = 1, 2, 3, 4, 5, 6$ and $x_1 = 0.5, x_2 = 1.5, x_3 = 2.5, x_4 = 3.5, x_5 = 4.5, x_6 = 5.5$. (i.e., $x_i = i - 0.5$)

(c) Please determine the probability $F_i = F(x_i)$, the mean, and the variance of the variable $x = x_i$, if $i = 1, 2, 3$ and $x_1 = 2, x_2 = 4, x_3 = 6$. (i.e., $x_i = 2i$)

(d) Please determine the probability $F_i = F(x_i)$, the mean, and the variance of the variable $x = x_i$, if $i = 1, 2, 3$ and $x_1 = 1, x_2 = 3, x_3 = 5$. (i.e., $x_i = 2(i - 0.5) = 2i - 1$)

(e) Please determine the probability $F_i = F(x_i)$, the mean, and the variance of the variable $x = x_i$, if $i = 1, 2, 3, \dots, 12$ and $x_i = i/2$.

(f) Please determine the probability $F_i = F(x_i)$, the mean, and the variance of the variable $x = x_i$, if $i = 1, 2, 3, \dots, 12$ and $x_i = (i - 0.5)/2$.

請將以上「計算結果」填入下表中

	Case 1	Case 2a	Case 2b	Case 2c	Case 2d	Case 2e	Case 2f
$F_i = F(x_i)$	N/A						
mean							
variance							

由以上結果，請找出 Case 2a-2f 中 $F_i = F(x_i)$ 與 Case 1 中的 $f(x)$ 之間的關係，以及不同 Case 之平均值 (mean) 與變異數 (variance) 之間的關係。

(提示：用繪圖的方式標示個例子的機率大小或分佈情形或許可以幫助理解與作答)