Vector Analysis 直角坐標

$$d\boldsymbol{r} = \hat{x}dx + \hat{y}dy + \hat{z}dz$$

Volumn integration of a scalar field, such as the density field $\rho(\boldsymbol{r}) = \rho(x, y, z)$

$$\iiint [\rho(\boldsymbol{r})] d\boldsymbol{r} = \iiint [\rho(x, y, z)]\, d^3x = \iiint [\rho(x, y, z)]\, dx\, dy\, dz \qquad (1)$$

Vector differential operator

$$\nabla = \frac{\partial}{\partial \boldsymbol{r}} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

Given a scalar field $f(\boldsymbol{r}) = f(\vec{r}) = f(x, y, z)$, then the gradient of $f(\boldsymbol{r})$ is

$$\text{grad } f(\boldsymbol{r}) = \nabla f(\boldsymbol{r}) = \hat{x}\frac{\partial f(x, y, z)}{\partial x} + \hat{y}\frac{\partial f(x, y, z)}{\partial y} + \hat{z}\frac{\partial f(x, y, z)}{\partial z} \qquad (2)$$

Given a vector field $\boldsymbol{A}(\boldsymbol{r}) = \vec{A}(\vec{r}) = \hat{x}A_x(x, y, z) + \hat{y}A_y(x, y, z) + \hat{z}A_z(x, y, z)$, then the divergence of $\boldsymbol{A}(\boldsymbol{r})$ is

$$\text{div } \boldsymbol{A}(\boldsymbol{r}) = \nabla \cdot \boldsymbol{A}(\boldsymbol{r}) = \frac{\partial A_x(x, y, z)}{\partial x} + \frac{\partial A_y(x, y, z)}{\partial y} + \frac{\partial A_z(x, y, z)}{\partial z} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \qquad (3)$$

and the curl of $\boldsymbol{A}(\boldsymbol{r})$ is

$$\text{curl } \boldsymbol{A}(\boldsymbol{r}) = \nabla \times \boldsymbol{A}(\boldsymbol{r}) = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) \qquad (4)$$

Vector Analysis 柱面坐標

$$d\boldsymbol{r} = \hat{r}dr + \hat{\theta}rd\theta + \hat{z}dz$$

Volumn integration of a scalar field, such as the density field $\rho(\boldsymbol{r}) = \rho(r,\theta,z)$

$$\iiint [\rho(\boldsymbol{r})]d\boldsymbol{r} = \iiint [\rho(r,\theta,z)]\, d^3x = \iiint [\rho(r,\theta,z)]\, dr\, rd\theta\, dz \qquad (5)$$

Vector differential operator

$$\nabla = \frac{\partial}{\partial \boldsymbol{r}} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{z}\frac{\partial}{\partial z}$$

Given a scalar field $f(\boldsymbol{r}) = f(\vec{r}) = f(r,\theta,z)$, then the gradient of $f(\boldsymbol{r})$ is

$$\text{grad } f(\boldsymbol{r}) = \nabla f(\boldsymbol{r}) = \hat{r}\frac{\partial f(r,\theta,z)}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial f(r,\theta,z)}{\partial \theta} + \hat{z}\frac{\partial f(r,\theta,z)}{\partial z} \qquad (6)$$

Given a vector field $\boldsymbol{A}(\boldsymbol{r}) = \vec{A}(\vec{r}) = \hat{r}\, A_r(r,\theta,z) + \hat{\theta}\, A_\theta(r,\theta,z) + \hat{z}\, A_z(r,\theta,z)$, then the divergence of $\boldsymbol{A}(\boldsymbol{r})$ is

$$\text{div } \boldsymbol{A}(\boldsymbol{r}) = \nabla \cdot \boldsymbol{A}(\boldsymbol{r}) = \frac{\partial A_r}{\partial r} + \frac{A_r}{r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z} \qquad (7)$$

and the curl of $\boldsymbol{A}(\boldsymbol{r})$ is

$$\text{curl } \boldsymbol{A}(\boldsymbol{r}) = \nabla \times \boldsymbol{A}(\boldsymbol{r}) = \hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right) + \hat{\theta}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{z}\left(\frac{\partial A_\theta}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial \theta} + \frac{A_\theta}{r}\right) \qquad (8)$$

Vector Analysis 球面坐標

$$d\boldsymbol{r} = \hat{r}dr + \hat{\theta}\, r\, d\theta + \hat{\phi}\, r \sin\theta\, d\phi$$

Volumn integration of a scalar field, such as the density field $\rho(\boldsymbol{r}) = \rho(r, \theta, \phi)$

$$\iiint [\rho(\boldsymbol{r})] d\boldsymbol{r} = \iiint [\rho(r, \theta, \phi)]\, d^3x = \iiint [\rho(r, \theta, \phi)]\, dr\, r d\theta\, r \sin\theta\, d\phi = \iiint [\rho(r, \theta, \phi)]\, r^2 \sin\theta\ dr\, d\theta\, d\phi \qquad (9)$$

Vector differential operator

$$\nabla = \frac{\partial}{\partial \boldsymbol{r}} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}$$

Given a scalar field $f(\boldsymbol{r}) = f(\vec{r}) = f(r, \theta, \phi)$, then the gradient of $f(\boldsymbol{r})$ is

$$\text{grad } f(\boldsymbol{r}) = \nabla f(\boldsymbol{r}) = \hat{r}\frac{\partial f(r, \theta, \phi)}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial f(r, \theta, \phi)}{\partial \theta} + \hat{\phi}\frac{1}{r \sin\theta}\frac{\partial f(r, \theta, \phi)}{\partial \phi} \qquad (10)$$

Given a vector field $\boldsymbol{A}(\boldsymbol{r}) = \vec{A}(\vec{r}) = \hat{r}\, A_r(r, \theta, \phi) + \hat{\theta}\, A_\theta(r, \theta, \phi) + \hat{\phi}\, A_\phi(r, \theta, \phi)$, then the divergence of $\boldsymbol{A}(\boldsymbol{r})$ is

$$\text{div } \boldsymbol{A}(\boldsymbol{r}) = \nabla \cdot \boldsymbol{A}(\boldsymbol{r}) = \frac{\partial A_r}{\partial r} + \frac{2A_r}{r} + \frac{\cos\theta\, A_\theta}{r \sin\theta} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{1}{r \sin\theta}\frac{\partial A_\phi}{\partial \phi} \qquad (11)$$

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and the curl of $\boldsymbol{A}(\boldsymbol{r})$ is

$$\text{curl } \boldsymbol{A}(\boldsymbol{r}) = \nabla \times \boldsymbol{A}(\boldsymbol{r}) = \hat{r}\left(\frac{1}{r}\frac{\partial A_\phi}{\partial \theta} - \frac{1}{r \sin\theta}\frac{\partial A_\theta}{\partial \phi} + \frac{\cos\theta\, A_\phi}{r \sin\theta}\right) + \hat{\theta}\left(\frac{1}{r \sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial A_\phi}{\partial r} - \frac{A_\phi}{r}\right) + \hat{\phi}\left(\frac{\partial A_\theta}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial \theta} + \frac{A_\theta}{r}\right) \qquad (12)$$