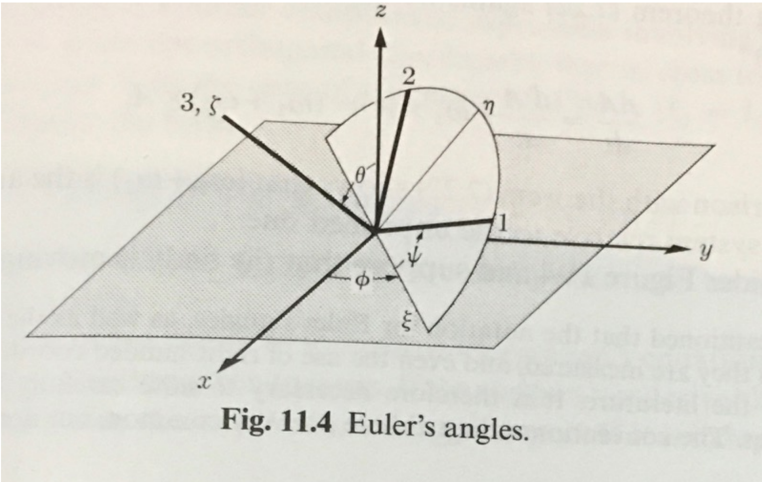
https://www.youtube.com/watch?v=q0jgqeS_ACM
Euler Angles - Interactive 3D Graphics

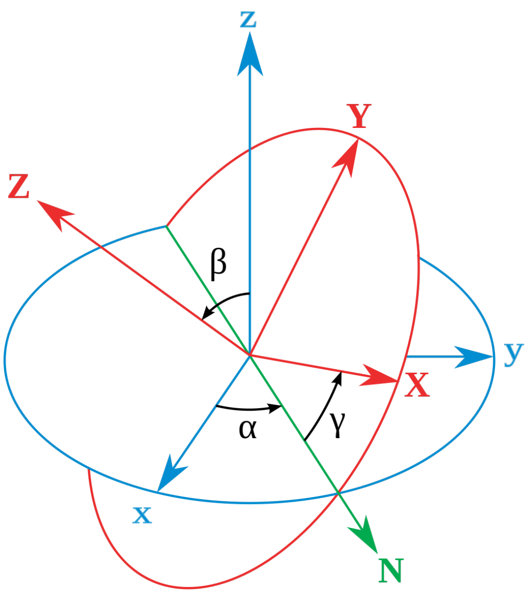
飛機			衛星、行星 運動
Yaw/Head angle 偏轉角	ϕ	α	precession 公轉面進動
Pitch angle 俯仰角	θ	β	nutation 章動 inclination angle 公轉面傾角
Roll angle 翻滾角	ψ	γ	intrinsic rotation 公轉角度 angle of revolution
	ξ	N	ascending node 升交點

注意它們的時間尺度 Time scales 通常為 $T_\gamma < T_\beta < T_\alpha$

https://en.wikipedia.org/wiki/Euler_angles
Euler angles



此圖由課本下載 (Symon, 1960)



此圖下載自維基百科

Proper Euler angles geometrical definition. The xyz (fixed) system is shown in blue, the XYZ (rotated) system is shown in red. The line of nodes (N) is shown in green

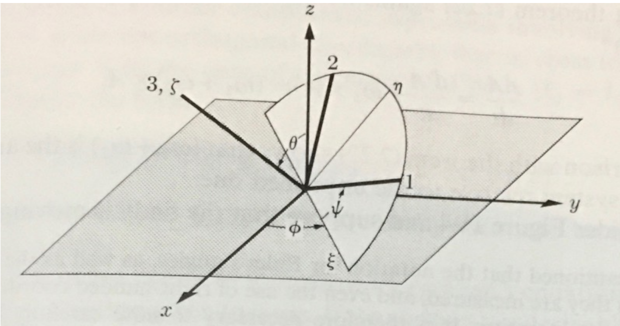


Fig. 11.4 Euler's angles.

We may therefore express $\boldsymbol{\omega}$ in terms of its components along the principal axes:

$$\omega_1 = \dot{\theta}\cos\psi + \dot{\phi}\sin\theta\sin\psi,$$
$$\omega_2 = -\dot{\theta}\sin\psi + \dot{\phi}\sin\theta\cos\psi, \quad (11\text{–}44)$$
$$\omega_3 = \dot{\psi} + \dot{\phi}\cos\theta.$$

The kinetic energy is now given by Eq. (10–153):

$$T = \tfrac{1}{2}I_1\omega_1^2 + \tfrac{1}{2}I_2\omega_2^2 + \tfrac{1}{2}I_3\omega_3^2. \quad (11\text{–}45)$$

The kinetic energy is a rather complicated expression involving $\dot{\theta}$, $\dot{\phi}$, $\dot{\psi}$, θ, and ψ. Note that θ, ϕ, ψ are not orthogonal coordinates, i.e., cross terms involving $\dot{\theta}\dot{\phi}$ and $\dot{\psi}\dot{\phi}$ appear in T. In the case of a symmetrical body ($I_1 = I_2$), the expression for T simplifies to the form:

$$T = \tfrac{1}{2}I_1\dot{\theta}^2 + \tfrac{1}{2}I_1\dot{\phi}^2\sin^2\theta + \tfrac{1}{2}I_3(\dot{\psi} + \dot{\phi}\cos\theta)^2. \quad (11\text{–}46)$$

The generalized forces Q_θ, Q_ϕ, Q_ψ are easily shown to be the torques about the ξ-, z-, and 3-axes.

根據維基百科那張圖，可知轉動的順序

1. 先以 $\hat{z}$ 軸轉 ϕ 角
2. 再以 $\hat{\xi}$ 軸轉 θ 角
3. 再以 $\hat{\zeta}$ 軸 ($\hat{e}_3$ 軸)轉 ψ 角

角速度為$\vec{\omega}$

$$\vec{\omega} = \dot{\phi}\hat{z} + \dot{\theta}\hat{\xi} + \dot{\psi}\hat{e}_3 \quad (1)$$

where

$$\hat{z} = \cos\theta\,\hat{e}_3 + \sin\theta\,\hat{\eta} \quad (2)$$
$$\hat{\eta} = \cos\psi\,\hat{e}_2 + \sin\psi\,\hat{e}_1 \quad (3)$$
$$\hat{\xi} = \hat{\eta}\times\hat{e}_3 = \cos\psi\,\hat{e}_1 - \sin\psi\,\hat{e}_2 \quad (4)$$

Substituting Equation (3) into Equation (2) to eliminate $\hat{\eta}$, then substituting the resulting equation into Equation (1) to eliminate $\hat{z}$, and then substituting Equation (4) into Equation (1) to eliminate $\hat{\xi}$, it yields

$$\vec{\omega} = \dot{\phi}[\cos\theta\,\hat{e}_3 + \sin\theta\,(\cos\psi\,\hat{e}_2 + \sin\psi\,\hat{e}_1)] + \dot{\theta}(\cos\psi\,\hat{e}_1 - \sin\psi\,\hat{e}_2) + \dot{\psi}\hat{e}_3 \quad (5)$$

or

$$\vec{\omega} = \omega_1\hat{e}_1 + \omega_2\hat{e}_2 + \omega_2\hat{e}_3 \quad (6)$$

and

$$\omega_1 = \dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi \quad (7)$$
$$\omega_2 = \dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi \quad (8)$$
$$\omega_3 = \dot{\phi}\cos\theta + \dot{\psi} \quad (9)$$

The kinetic energy is

$$T = \frac{1}{2}\vec{\omega}\cdot\vec{\vec{I}}\cdot\vec{\omega} = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 + \frac{1}{2}I_3\omega_3^2 \quad (10)$$

For $I_1 = I_1$, the kinetic energy becomes

$$T = \frac{1}{2}I_1(\omega_1^2 + \omega_2^2) + \frac{1}{2}I_3\omega_3^2 = \frac{1}{2}I_1\left(\dot{\phi}^2\sin^2\theta + \dot{\theta}^2\right) + \frac{1}{2}I_3\left(\dot{\phi}\cos\theta + \dot{\psi}\right)^2 \quad (11)$$

Namely,

$$T = T(\theta, \dot{\phi}, \dot{\theta}, \dot{\psi})$$

If the potential energy is only a function of θ, then p_ϕ and p_ψ are constants of motion.

有關講一中所介紹的第一個影片中所提到的 gimbal lock 問題，很多同學有疑問。為此，陳宥旻同學推薦了一個影片

https://www.youtube.com/watch?v=zc8b2Jo7mno&t=216s

Euler (gimbal lock) Explained

完整影片連結

https://www.youtube.com/watch?v=zc8b2Jo7mno

這個影片說明，當我們調整或遙控一個鏡頭、無人機、或軟體中的 3D 旋轉時，需要注意的 gimbal lock（環向鎖定 or 萬向鎖）的問題。這是一個數學問題，不是物理問題。所以這門力學課，只會介紹這些名詞，不會深入探討它。但是各位如果想玩遙控鏡頭、無人機、或軟體中的 3D 旋轉前，最好先了解一下這個問題。

不過，就我玩天文望遠鏡的經驗，其實只要兩個旋轉軸，就可以旋轉到你想要轉到的方向了。因此就算發生 gimbal lock，應該也可以很容易的解鎖。