

Chain Rule

Let $f(x) = (3x + 5)^3 + (3x + 5)^2 + 2x + 5$

Find $f'(x) = df(x)/dx = ?$

要解答這個問題有兩種方法。一種笨方法，一種取巧方法。

Method 1 (笨方法) :

$$\begin{aligned} f(x) &= (3x + 5)^3 + (3x + 5)^2 + 2x + 5 \\ &= (27x^3 + 135x^2 + 225x + 125) + (9x^2 + 30x + 25) + 2x + 5 \\ &= 27x^3 + 144x^2 + 257x + 155 \end{aligned}$$

Thus,

$$f'(x) = 81x^2 + 288x + 257$$

Let $f(x) = (3x + 5)^3 + (3x + 5)^2 + 2x + 5$

Find $f'(x) = df(x)/dx = ?$

Method 2 (取巧方法) :

Let $g(x) = 3x + 5$

Then

$$f(x) = F[g(x), x] = g^3 + g^2 + g - x$$

Thus,

$$\begin{aligned} f'(x) &= \frac{dF[g(x), x]}{dx} = \frac{\partial F}{\partial g} \frac{dg}{dx} + \frac{\partial F}{\partial x} = (3g^2 + 2g + 1)(3) - 1 \\ &= [3(3x + 5)^2 + 2(3x + 5) + 1](3) - 1 \\ &= 9(9x^2 + 30x + 25) + 18x + 33 - 1 \\ &= 81x^2 + 288x + 257 \end{aligned}$$

結果與第一個笨方法相同 $f'(x) = 81x^2 + 288x + 257$

其實，兩種方法都做一做，也是個好主意，可以double check your results!

$$\begin{aligned} \text{For } F[g(x), x] \\ \frac{\partial F}{\partial g} &= \left[\frac{\partial F}{\partial g} \right]_{x=\text{const.}} \\ \frac{\partial F}{\partial x} &= \left[\frac{\partial F}{\partial x} \right]_{g=\text{const.}} \end{aligned}$$

Chain Rule

In cylindrical coordinate system

$$\frac{d\hat{r}[\theta(t)]}{dt} = \frac{d\hat{r}}{d\theta} \frac{d\theta}{dt}$$

Likewise, for $d\hat{\theta}[\theta(t)]/dt$

In spherical coordinate system

$$\frac{d\hat{r}[\theta(t), \phi(t)]}{dt} = \frac{\partial \hat{r}}{\partial \theta} \frac{d\theta}{dt} + \frac{\partial \hat{r}}{\partial \phi} \frac{d\phi}{dt}$$

Likewise, for $d\hat{\theta}[\theta(t), \phi(t)]/dt$ and $d\hat{\phi}[\phi(t)]/dt$