Lecture 5. The Inner Magnetosphere

5.1. Co-rotating E-field

A magnetohydodynamic (MHD) plasma is a simplified plasma model at low-frequency and long-wavelength limit. Consider time scale much longer than the Alfven wave traveling time along the magnetic field line, we can assume that a magnetized plasma system satisfies the MHD Ohm's law, that is

$$\mathbf{E} = -\nabla\phi = -\mathbf{V}\times\mathbf{B}.$$

Thus, for the MHD plasma, both magnetic field lines and streamlines are equal potential lines. Namely, if a perpendicular electric field (i.e., E-field which is perpendicular to the local magnetic field) is generated at one end of the magnetic field line, an Alfevn wave will carry this information and propagate along the magnetic field line to make the electric potential to be constant along the magnetic field line. Likewise, the fast mode wave in the MHD plasma will carry the electric field information along the streamlines to make the streamlines in the MHD plasma to be equal-potential lines.

We are going to show in this section that if the magnetic field in the inner magnetosphere of the Earth satisfies the dipole magnetic field model, then plasma confined by the dipole magnetic field will co-rotate with the Earth due to the presence of a co-rotating electric field. From the dipole field configuration, we can estimate the perpendicular electric field distribution along the magnetic field line based on the electric field generated in the ionosphere.

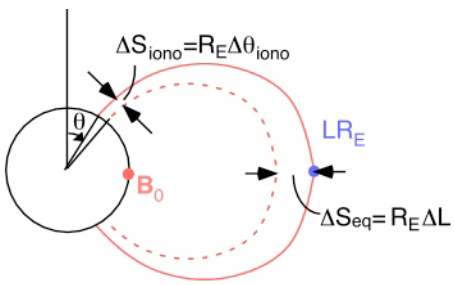

Figure 5.1. A sketch of the separation distance between two magnetic field lines along an L-value dipole magnetic field line.

The equation for dipole magnetic field line can be written as

$$r = r_{eq} \sin^2 \theta(r) \tag{5.1}$$

Figure 5.1 sketches the separation distance between two magnetic field lines along an L-value dipole magnetic field line. The solid curve in Figure 5.1 is a dipole magnetic field line, which passes magnetic equatorial plane at $r_{eq} = LR_E$. Let this L-value field (solid curve) pass ionosphere at point $(r,\theta) \approx (R_E, \theta_{iono})$. Eq. (5.1) yields

$$R_E = LR_E \sin^2[\theta(r = R_E)] = LR_E \sin^2 \theta_{iono}$$

or

$$L \sin^2 \theta_{iono} = 1 \tag{5.2}$$

Differentiating Eq. (5.2) once, it yields,

$$\frac{\Delta L}{L} + \frac{2\Delta \sin\theta_{iono}}{\sin\theta_{iono}} = 0$$

or

$$\Delta L = -L \frac{2\cos\theta_{iono} \Delta\theta_{iono}}{\sin\theta_{iono}} \tag{5.3}$$

The dash curve in Figure 5.1 is a dipole magnetic field line, which passes magnetic equatorial plane at $r_{eq} = (L + \Delta L)R_E$, where $\Delta L < 0$. Let this field line pass ionosphere at $(r,\theta) = (R_E, \theta_{iono} + \Delta\theta_{iono})$. The distance between the solid curve and the dash curve at the equatorial plane is

$$\Delta S_{eq} = -R_E \Delta L \tag{5.4}$$

where $\Delta L < 0$. The distance between the solid curve and the dash curve in the ionosphere is approximately

$$\Delta S_{iono} \approx R_E \Delta\theta_{iono} \tag{5.5}$$

Substituting Eq. (5.3) into Eq. (5.4) to eliminate ΔL, then substituting Eq. (5.5) into the resulting equation to eliminate $\Delta\theta_{iono}$, it yields

$$\Delta S_{eq} = -R_E \Delta L = -(-L \frac{2\cos\theta_{iono}\Delta\theta_{iono}}{\sin\theta_{iono}}) R_E = \frac{2\cos\theta_{iono}}{\sin^3\theta_{iono}} \Delta S_{iono}$$

or

$$\frac{\Delta S_{eq}}{\Delta S_{iono}} = \frac{2\cos\theta_{iono}}{\sin^3\theta_{iono}} \tag{5.6}$$

The electric field generated at the ionosphere is

$$\begin{aligned}\mathbf{E}_{iono} = -\mathbf{V}_{iono} \times \mathbf{B}_{iono} &= -(\omega_E R_E \sin\theta_{iono}\, \hat{\phi}) \times (-B_0 2\cos\theta_{iono} \hat{r} - B_0 \sin\theta_{iono} \hat{\theta}) \\ &= +\hat{\theta}\omega_E R_E \sin\theta_{iono} B_0 2\cos\theta_{iono} - \hat{r}\omega_E R_E \sin^2\theta_{iono} B_0\end{aligned} \tag{5.7}$$

where ω_E is the angular velocity of Earth rotating and B_0 is the strength of the dipole magnetic field at the equator of Earth's surface as indicated in Figure 5.1. The potential jump between the solid curve and the dash curve is

$$\Delta\phi = \mathbf{E}_{iono} \cdot (\hat{\theta}\Delta S_{iono}) = \omega_E R_E \sin\theta_{iono} B_0 2\cos\theta_{iono}\Delta S_{iono} \tag{5.8}$$

Since the magnetic field lines are equal potential lines, the potential jump between the solid curve and the dash curve can also be written as

$$\Delta\phi = \mathbf{E}_{eq} \cdot (-\hat{r})\Delta S_{eq}$$

i.e.,

$$\mathbf{E}_{eq} = (-\hat{r})\frac{\Delta\phi}{\Delta S_{eq}} \tag{5.9}$$

Substituting Eq. (5.8) into Eq. (5.9) to eliminate $\Delta\phi$, and then substituting Eq. (5.6) into the resulting equation to eliminate $\Delta S_{iono}/\Delta S_{eq}$, it yields

$$\mathbf{E}_{eq} = (-\hat{r})(\omega_E R_E \sin\theta_{iono} B_0 2\cos\theta_{iono})/(\frac{2\cos\theta_{iono}}{\sin^3\theta_{iono}}) = (-\hat{r})\omega_E R_E B_0 \sin^4\theta_{iono} \tag{5.10}$$

Substituting Eq. (5.2) into Eq. (5.10) to eliminate $\sin^4\theta_{iono}$, it yields

$$\mathbf{E}_{eq} = -\hat{r}\frac{\omega_E R_E B_0}{L^2} \tag{5.11}$$

The plasma flow velocity in the equatorial plane can be estimated from the $\mathbf{E}\times\mathbf{B}$ drift velocity, i.e.,

$$\mathbf{V}_{eq} = \frac{\mathbf{E}_{eq}\times\mathbf{B}_{eq}}{(B_{eq})^2} = (-\hat{r}\frac{\omega_E R_E B_0}{L^2})\times(-\hat{\theta}\frac{B_0}{L^3})/(\frac{B_0}{L^3})^2 = \hat{\phi}\omega_E R_E L = \hat{\phi}\omega_E r_{eq} \tag{5.12}$$

Namely, plasma at the equatorial plane moves at the same angular velocity as the solid Earth. Electric field obtained in Eq. (5.11) is called the *co-rotating E-field* in the inner magnetosphere.

Exercise 5.1

It has been shown in this Lecture that the co-rotating electric field in the Earth's inner magnetosphere is perpendicular to the local magnetic field and is characterized by a negative radial component. Show that the co-rotating electric field in the Jovian magnetosphere is perpendicular to the local magnetic field and is characterized by a positive radial component.

5.2. Plasmasphere, Plasmapause, and Alfven Layer

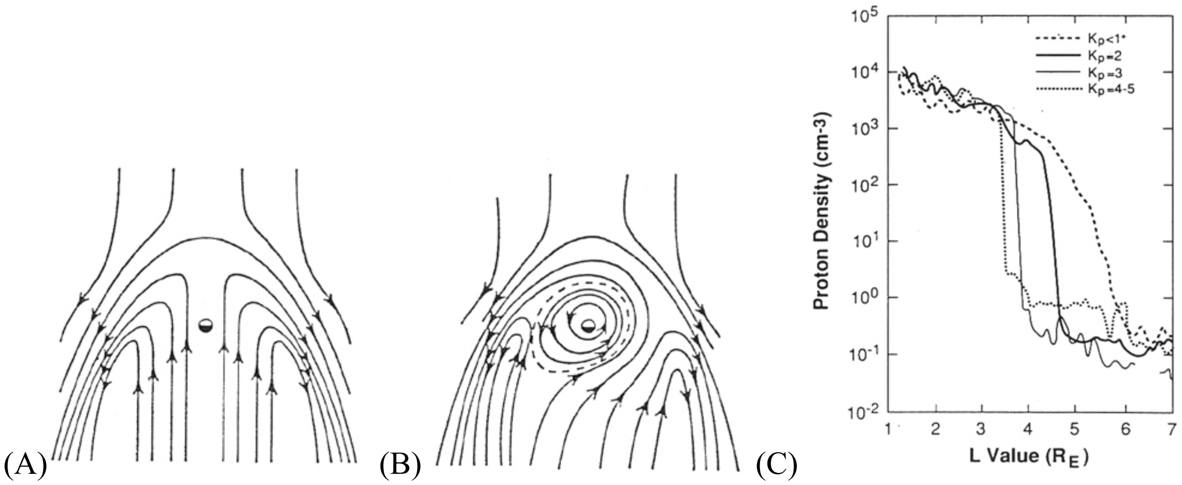

Figure 5.2.

（A）表示不考慮地球自轉效應，只考慮太陽風吹過磁尾兩側，在磁層赤道面上造成的環流結構。（B）表示若加上地球自轉效應，則在靠近地球附近的磁層赤道面上，會產生一圈電漿環流，繞著地球兜圈子，而不會流失到廣大的磁尾空間中。圖中虛線圈內的區域，即為電漿球層在赤道面上的投影。（C）顯示在不同Kp時，電漿球層內外電漿密度分布情形。密度梯度最大處為電漿球層頂。圖中顯示電漿球層頂的位置，隨著Kp增加而內移，梯度也增加。

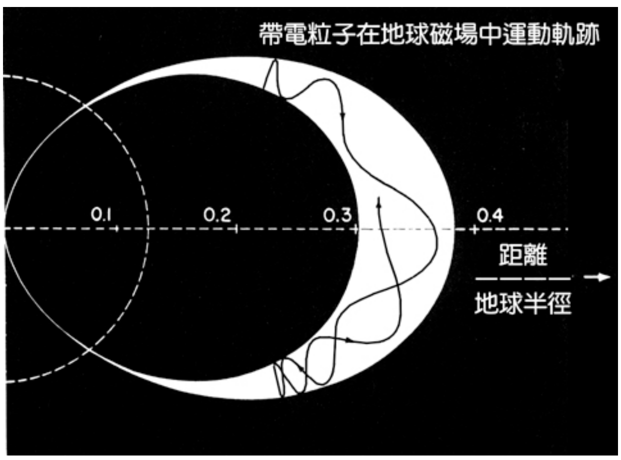

Figure 5.3. Bounce motion (or mirror motion)

Figure 5.4. Grad-B drift and curvature drift motion

5.3. Radiation Belts and Ring Current

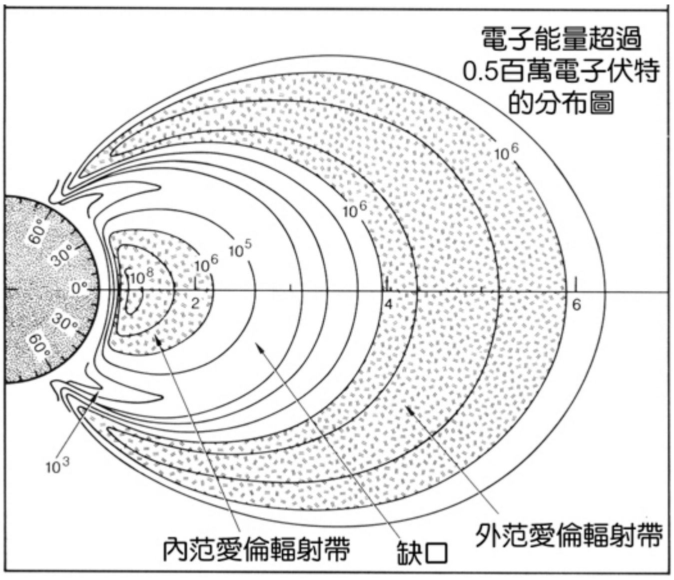

Figure 5.5. 內、外范愛倫輻射帶的空間分布情形。

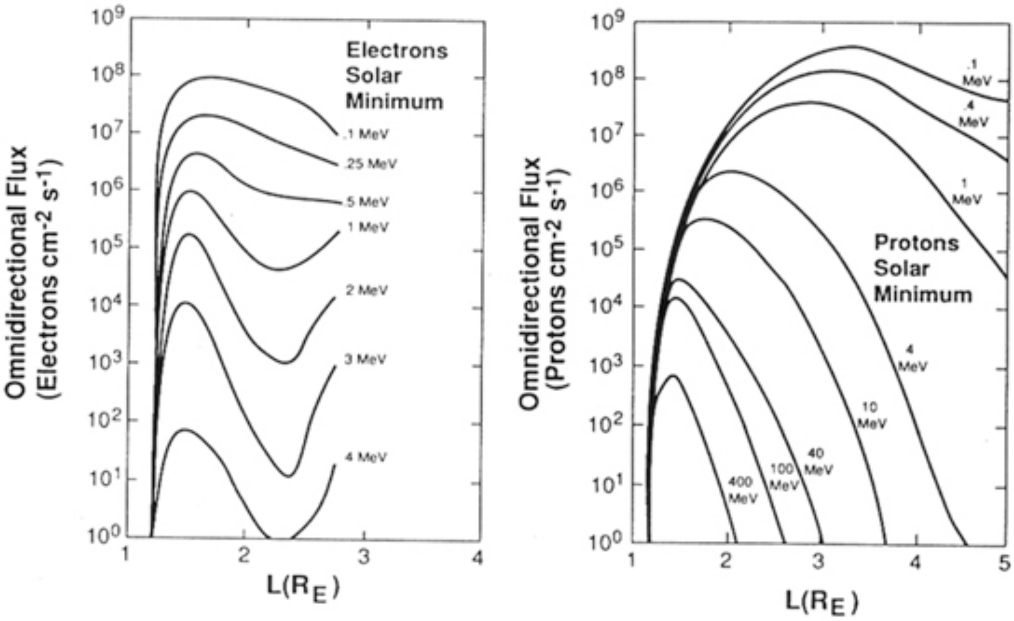

Figure 5.6.
范愛倫輻射帶中的高能電子與高能質子，在不同 L 的磁場線上的分布情形。內、外范愛倫輻射帶的分野可由高能電子的分布圖看出來。其中， L = 4 的磁場線，表示該磁場線通過磁赤道時，距離地心 4 個地球半徑。