## Lecture 2. Dipole-Axis-Dependent Coordinate Systems and Near Earth's Magnetic Field

## 2.1. Dipole-Axis-Dependent Coordinate Systems

## Exercise 2.1.

Please read the Appendix 3 in the following textbook

*Introduction to Space Physics*, edited by M. G. Kivelson and C. T. Russell, Cambridge University Press, 1995.

Find the definition of the GSM coordinate system and the SM coordinate system.

Answer of Exercise 2.1:

GSM Coordinate System (good for studying outer magnetosphere)

Geocentric Solar Magnetic System (A.3.3.6)

x-z plane contains Earth magnetic dipole and Sun-Earth line

 $\hat{x}$ : A unit vector pointing toward the Sun from the Earth.

- $\hat{z}$ : A unit vector perpendicular to the x-axis, roughly in the northward direction.
- $\hat{y}$ : A dawn-to-dusk unit vector. ( $\hat{y} = \hat{z} \times \hat{x}$ )

SM Coordinate System (good for studying inner magnetosphere, i.e., plasmasphere)

Solar Magnetic Coordinates (A.3.3.7)

x-z plane contains Earth magnetic dipole and Sun-Earth line

 $\hat{z}$ : A unit vector along Earth magnetic dipole, roughly in the northward direction.

 $\hat{x}$ : A unit vector perpendicular to the *z*-axis, roughly in the Earth-to-Sun direction.

 $\hat{y}$ : A dawn-to-dusk unit vector. ( $\hat{y} = \hat{z} \times \hat{x}$ )

# Exercise 2.2.

Let us consider the basis of the GSM coordinate system  $\{\hat{x}_{GSM}, \hat{y}_{GSM}, \hat{z}_{GSM}\}$ 

- (a) What is the "dawn-to-dusk direction"?
- (b) What is the "sunward direction"?
- (c) What is the "anti-sunward direction"?
- (d) What is the "tailward direction"?
- (e) What is the direction of the solar wind?

### 2.2. Coordinate Transformation

The detailed discussion on the coordinate transformations of a first-rank tensor (i.e., vector) and the coordinate transformation of a second-rank tensor, such as a thermal pressure tensor can be found in the Appendix A.

### Exercise 2.3.

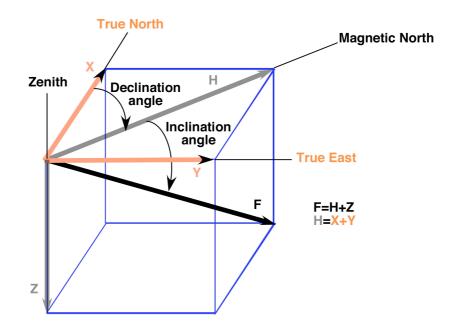
If we define the dipole titling angle in the GSM coordinate system to be the angle between  $\hat{z}_{GSM}$  and  $\hat{z}_{SM}$ . A positive titling angle indicates that  $\hat{z}_{SM}$  is titled toward the Sun. If the dipole titling angle in the GSM coordinate system is  $\alpha$ , find out the coordinate transform matrix **A**, which can transfer a vector field **V** from the SM coordinate system,  $(\mathbf{V})_{SM} = (U_x, U_y, U_z)$ , to the GSM coordinate system,  $(\mathbf{V})_{GSM} = (V_x, V_y, V_z)$ , i.e.,

$$(\mathbf{V})_{GSM} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \mathbf{A}(\mathbf{V})_{SM} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} U_x \\ U_y \\ U_z \end{bmatrix}$$

### 2.3. Near Earth Magnetic Field

The dipole magnetic field model is only applicable to the region roughly between 2 to 5 Earth radii from the Earth center. In the region far from the Earth, the Earth magnetic field is controlled by the current system in the boundary layers in the outer magnetosphere. In the region very close to the Earth surface, the crust magnetic fields are the important sources of the background magnetic field. One can find to the detailed surface magnetic field distribution in following webpage, <u>http://www.ngdc.noaa.gov/geomag/WMM/image.shtml</u> where the D, F, H, I, X (true north), Y (true east), Z components are described in Figure 2.1. The definition of the seven parameter can also be found in the following web page. http://www.ngdc.noaa.gov/geomag/WMM/soft.shtml

The crustal field in the mid-ocean ridge in the Atlantic Ocean yields very weak magnetic field in the South Atlantic Anomaly (SAA) area, which can be seen as a minimum magnetic field in the Main Field Total Intensity (F) plot shown in the NOAA -US/UK World Magnetic Model -- Epoch 2010 plot. Energetic charge particles are trapped in the SAA weak magnetic field region. More information on SAA can be found by searching the key words "South Atlantic Anomaly" on google.

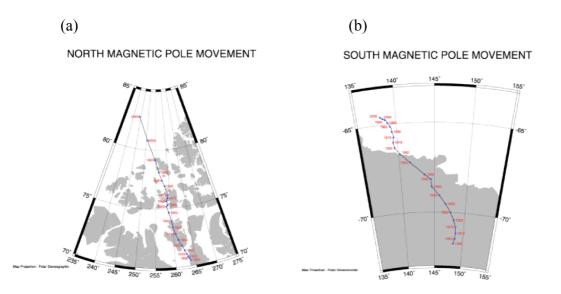


**Figure 2.1.** shows the seven parameters D, F, H, I, X, Y, and Z, commonly used to describe the surface magnetic field. The year 2010 surface magnetic field information can be found at <a href="http://www.ngdc.noaa.gov/geomag/WMM/image.shtml">http://www.ngdc.noaa.gov/geomag/WMM/image.shtml</a>

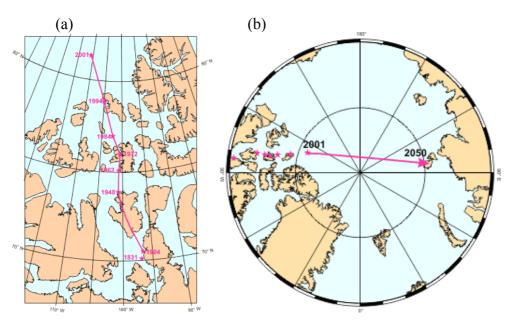
In addition to the crustal field, the orientation of the Earth magnetic dipole axis also keeps changing all the time.

Figure 2.2 shows observed magnetic-pole movement during years 1909-2005, where Panel (a) shows the north magnetic pole movement and Panel (b) shows the south magnetic pole movement. Note that, during the past 100 years, the north magnetic pole moves from lower latitude to higher latitude. But the south magnetic pole moves from higher latitude to lower latitude. The two images are downloaded from the following websites <a href="http://ftp.ngdc.noaa.gov/geomag/images/N\_magpl.gif">http://ftp.ngdc.noaa.gov/geomag/images/N\_magpl.gif</a> and <a href="http://ftp.ngdc.noaa.gov/geomag/images/S\_magpl.gif">ftp://ftp.ngdc.noaa.gov/geomag/images/S\_magpl.gif</a>.

Figure 2.3 shows the north magnetic pole movement during the years 1831-2001 and predicted movement of the north magnetic pole between years 2001-2050. (Information adapted from <a href="http://cgc.rncan.gc.ca/geomag/nmp/long\_mvt\_nmp\_e.php">http://cgc.rncan.gc.ca/geomag/nmp/long\_mvt\_nmp\_e.php</a>) (Credit: Geological Survey of Canada)



**Figure 2.2.** Observed magnetic-pole movement during years 1909-2005, where Panel (a) shows the north magnetic pole movement and Panel (b) shows the south magnetic pole movement. Note that, during the past 100 years, the north magnetic pole moves from lower latitude to higher latitude. But the south magnetic pole moves from higher latitude to lower latitude. The two images are downloaded from the following websites <a href="http://ftp.ngdc.noaa.gov/geomag/images/N\_magpl.gif">http://ftp.ngdc.noaa.gov/geomag/images/N\_magpl.gif</a> and <a href="http://ftp.ngdc.noaa.gov/geomag/images/S\_magpl.gif">ftp://ftp.ngdc.noaa.gov/geomag/images/S\_magpl.gif</a>.



**Figure 2.3.** The north magnetic pole movement during the years 1831-2001 and predicted movement of the north magnetic pole between years 2001-2050. (Information adapted from <a href="http://cgc.rncan.gc.ca/geomag/nmp/long\_mvt\_nmp\_e.php">http://cgc.rncan.gc.ca/geomag/nmp/long\_mvt\_nmp\_e.php</a> and <a href="http://science.nasa.gov/science-news/science-at-nasa/2003/29dec\_magneticfield/">http://science.nasa.gov/science-news/science-at-nasa/2003/29dec\_magneticfield/</a>)

Listed below are websites that can help you to learn more about the surface magnetic field near the Earth's surface.

- One can find an estimated value of local magnetic declination of a given location from the following website, <u>http://www.ngdc.noaa.gov/geomagmodels/Declination.jsp</u>
- (2) The detailed north magnetic pole movement during the years 2001-2005 can be found in the following websites, <u>http://cgc.rncan.gc.ca/geomag/nmp/northpole\_e.php</u> and <u>http://en.wikipedia.org/wiki/South\_Magnetic\_Pole</u>

North Magnetic Pole

(2001) 81.3°N 110.8°W

(2004 est) 82.3°N 113.4°W

(2005 est) 82.7°N 114.4°W

South Magnetic Pole

(1998) 64.6°S 138.5°E.

(2004 est) 63.5°S 138.0°E

(2008 est) 65°S 138°E

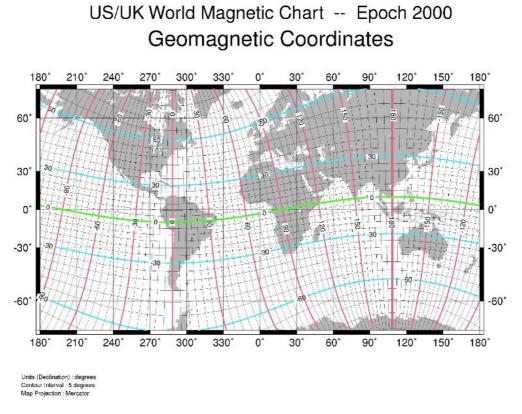
- (3) The possible north magnetic pole movement before year 1831 can be found in the following website <u>http://cgc.rncan.gc.ca/geomag/nmp/long\_mvt\_nmp2\_e.php</u>
- (4) More information on geomagnetic field and indeices can be found from the following website, <u>http://www.ngdc.noaa.gov/geomag/</u>
- (5) More information on space weather can be found from the following website, <u>http://spaceweather.com/</u>
- (6) More information on solar flare can be found from the following websites, <u>http://spaceweather.com/glossary/flareclasses.html</u>
  <u>http://hesperia.gsfc.nasa.gov/sftheory/flare.htm</u>
  <u>http://hesperia.gsfc.nasa.gov/sftheory/studyflare.htm</u>
  <u>http://hesperia.gsfc.nasa.gov/sftheory/cme.htm</u>
  ..., etc.

Here are some old information on the Earth's surface magnetic field distributions.

Figures 2.4 shows contours of geomagnetic coordinates in year 2001. The contours of the geomagnetic latitudes and longitudes can be obtained based on the given geographic latitude and longitude of the dipole axis shown in Table 2.1.

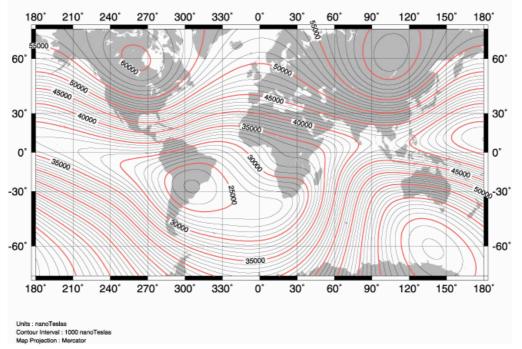
Figure 2.5 shows contour plot of the total geomagnetic field strength on Earth surface in year 2001. As we can see that the minimum of the field strength is not parallel to the geomagnetic equator, which is the green curve in Figure 2.2. A minimum of the magnetic field strength can be found in the South American and the South Atlantic Ocean. Ionosphere plasma often shows south Atlantic anomaly (SSA) in this minimum B region.

Figure 2.6 shows the contour plots of declination angle and inclination angle of geomagnetic field in year 2001. The declination angle and inclination angle provide useful information for ground observations. It can help us to determine the average magnetic field-aligned direction at different locations.



**Figure 2.4.** Contours of geomagnetic coordinates in year 2001. (Source: http://www.ngdc.noaa.gov/seg/potfld/faqgeom.shtml)





**Figure 2.5.** Contours of total geomagnetic field strength on Earth surface in year 2001. (Source: http://www.ngdc.noaa.gov/seg/potfld/faqgeom.shtml)

