

HW#5

Due 2023/12/04

1. Derive the equation of state.

$$\frac{\partial P}{\partial t} = -\bar{u} \cdot \nabla P - \gamma P \nabla \cdot \bar{u}$$

Hint: starting from $\frac{d}{dt} \ln(P \rho^{-\gamma}) = 0$ and combining with continuity equation.

2. Derive

$$\begin{bmatrix} \omega^2 - V_A^2 k_{\parallel}^2 - (C_S^2 + V_A^2) k_{\perp}^2 & 0 & -C_S^2 k_{\parallel} k_{\perp} \\ 0 & \omega^2 - V_A^2 k_{\parallel}^2 & 0 \\ -C_S^2 k_{\parallel} k_{\perp} & 0 & \omega^2 - C_S^2 k_{\parallel}^2 \end{bmatrix} \begin{pmatrix} u_{1x} \\ u_{1y} \\ u_{1z} \end{pmatrix} = 0$$

from the linearized MHD equations in the (ω, \bar{k}) domain by setting the background magnetic field in the z direction and wave vector on the xz plane. V_A is the Alfvén speed and C_S is the sound speed.